Noncooperative Game Theory for Resources Scheduling and Planning in Renewable Energy Communities

by

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Philosophiae Doctor in Sciences

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Abstract

The events of the last few years, such as the COVID-19 pandemic and the geopolitical crisis which started in Eastern Europe in early 2022, have thrown the natural gas and electricity European markets into an unprecedented crisis. These circumstances have highlighted the need for structural and regulatory measures to protect end-users from market fluctuations, while accelerating the transition to more resilient and sustainable systems. At the same time, the electricity sector is undergoing a profound transformation, with the rise of distributed energy resources and the growing adoption of decentralized solutions such as local solar and wind generation or individual storage systems. The developments reflect a paradigm shift towards more participatory and smart, consumer-centric energy models. In this context, renewable energy communities are emerging as key actors in the energy transition, and have received particular interest from economic, political and academic sectors in recent years. They are organized entities, gathering consumers and prosumers allowed to exchange renewable electricity produced locally without resorting to the traditional wholesale/retail markets. Their purpose is to provide economic, environmental or social benefits to the members and society, rather than to make a financial profit.

The main objective of this thesis is to model renewable energy communities and the various challenges surrounding them using noncooperative game theory. For that purpose, this work is divided into two parts, exploring a specific problem that can be modeled with a specific noncooperative game form.

In the first part of the thesis, we study local energy communities composed by end-users connected to the public electricity distribution network and sharing common resources such as the grid and their own local generation. We propose two market designs for the optimal day-ahead scheduling of energy exchanges within these communities. The first one implements a collaborative demand-side management scheme inside a community where members objectives are coupled through grid tariffs, the second allows the valuation of excess generation

in the community and on the retail market. Two grid tariff structures are tested, one academic and one which reflects the current Belgian regulations in terms of grid tariffs. Individuals' bills are obtained through 4 methods of cost allocation. Both designs are formulated as optimization problems first, and as noncooperative strategic games then. In the latter case, the existence and efficiency of the corresponding (generalized) Nash equilibria are studied and solution algorithms are proposed. The models are tested on a use-case made of 55 members and compared with a benchmark situation where members act individually. We compute the global renewable energy community and members' individual costs, study the inefficiencies of the decentralized models compared to social optima, and calculate technical indices such as self-consumption or peak-to-average ratio. In addition, we investigate the influence of retail electricity prices on the daily operation of the energy community. A sensitivity analysis is performed on the retail electricity prices and we measure the impact on the total community and members individuals costs and interest in joining/leaving the community.

The second part focuses on the integration of a new member inside an existing renewable energy community. We propose two distinct approaches. In the first structure, we model the case of an external user interested in joining the community, with our without investment contribution. The second approach examines the situation where the community is the instigator of its own expansion. This allows us to analyze how the flexibility or thoroughness of integration processes can influence the community dynamics and its ability to remain consistent with its objectives. Long-term (investments and tariff adjustments) and short-term decisions (day-ahead resources scheduling) are handled by an extensive-form game taking into account the uncertainty linked to the evolution of the retail market price. In particular, we use the results obtained in the first part to model and solve the short-term level. We also include the case where potential candidates and the community present heterogeneous preferences, reflecting varied objectives and priorities, such as minimizing costs or CO_2 emissions, maximizing return on investments, etc. In addition, we compare the decision-making processes of candidate users and the community under uncertainty. Our analysis is based on two distinct theoretical frameworks: (1) expected utility theory, which assumes perfect rationality on the agents' side, and (2) prospect theory, which captures the bounded rationality of individuals and their biases in risk perception. The models developed were tested on three renewable energy communities with distinct energy profiles and a varied list of candidates. Different combinations of preference criteria and parameters of the decision functions are explored in order to analyze the interactions and impacts of agents' preferences and perceptions. The models developed in this part have a sufficiently general structure to be extended to other types of decisions and problems, as well as to a variety of stakeholder profiles.

Contents

List of Abbreviations

CDE	Carbon Dioxide Emissions.
CMP	Community Mismatch Profile.
CSC	Collective Self-Consumption.
DA	Day-Ahead.
DER	Distributed Energy Resource.
DSM	Demand-Side Management.
DSO	Distribution System Operator.
ESS	Energy Storage System.
EU	European Union.
EUT	Expected Utility Theory.
EV	Electric Vehicles.
GNE	Generalized Nash Equilibrium.
GNEP	Generalized Nash Equilibrium Problem.
HP	Heat Pump.
HV	High-Voltage.
KKT	Karush-Kuhn-Tucker.
KPI	Key Performance Indicator.
LICQ	Linear Independence Constraint Qualification.
LP	Linear Programming.
LT	Long-Term.
LV	Low Voltage.
MCP	Mixed Complementary Problem.
MS	Matching Score.

MV	Medium Voltage.
NE	Nash Equilibrium.
NEP	Nash Equilibrium Problem.
NMIP	New Member Integration Problem.
NPV	Net Present Value.
PAR	Peak to Average Ratio.
PDA	Proximal Decomposition Algorithm.
PG	Potential Game.
PoA	Price of Anarchy.
PoS	Price of Stability.
PT	Prospect Theory.
PV	Photovoltaic.
QP	Quadratic Programming.
QVI	Quasi-Variational Inequality.
REC	Renewable Energy Community.
ROI	Return on Investment.
SC	Social Cost.
SCR	Self-Consumption Rate.
SPE	Subgame Perfect Equilibrium.
SQ	Slater's Constraint Qualification.
SSR	Self-Sufficient Rate.
ST	Short-Term.
TSO	Transmission System Operator.
VCG	Vickrey-Clarke-Groves.
VE	Variational Equilibrium.
VI	Variational Inequality.

CHAPTER 1.

Introduction

1.1. Context and motivation

The events of the last few years, such as the COVID-19 pandemic and the geopolitical crisis, which started in Eastern Europe in early 2022, have thrown the natural gas and electricity European markets into an unprecedented crisis [1]. Retail electricity prices suffered from the same trend: the vast majority of the contract offer in the retail markets moved from fixed-price to variable-price contracts, for which the retail price is indexed on wholesale spot markets on a monthly or quarterly basis. Each member state has taken their own measure to protect end-users from the sharp increase in their energy costs, with significant variations between countries, whereas the debate on the relevance of marginal pricing for wholesale electricity markets resurfaced among policy-makers and the scientific community (see e.g., [2]). These circumstances have highlighted the necessity of establishing permanent structural and regulatory measures to protect end-users from market fluctuations, while accelerating the transition to more resilient and sustainable systems. Moreover, the growing global demand for energy is intensifying, placing significant pressure on Europe's energy systems. Europe's energy supply remains heavily dependent on imported fossil fuels, with certain countries relying extensively on Russian resources. The emergency to advance towards decarbonization has now gained an additional critical dimension: ensuring energy security and independence.

At the same time, the electricity sector is undergoing a profound transformation, with the rise of distributed energy resources (DERs) and the growing adoption of decentralized solutions, such as local solar and wind generation or individual storage systems. Technological advancements, combined with increasing environmental and ecological awareness among citizens, have facilitated the rise of prosumers—individuals who both produce and consume electricity. Prosumers have the ability to draw energy from or feed energy into the existing distribution network, actively supporting grid operations while reducing reliance on centralized power plants through self-consumption. Additionally, they can participate in demand-response initiatives by lowering their energy usage during peak periods. The European Union has widely acknowledged the critical role of prosumers in achieving its ambitious environmental goals and emphasizes the importance of empowering them to fully engage with the energy system. The developments reflect a paradigm shift towards more participatory and smart, prosumer-centric energy models.

In this context, *energy communities* [3, 4] are emerging as key actors in the energy transition, and have received particular interest from economic, political and academic sectors in recent years. We are especially focused on Renewable Energy Communities (RECs) in this work [5]. They are organized entities, gathering consumers and prosumers allowed to exchange renewable electricity produced locally without resorting to the traditional wholesale/retail markets. Their purpose is to provide economic, environmental or social benefits to members and society, rather than to make a financial profit. Introduced by the European Union (EU) Commission in its Directive 2018/2001 [6], they aim to 1) place the citizen at the center of the liberalized electricity supply chain, 2) stimulate local joint investment in renewable generation and storage assets, 3) unlock flexibility inherently present in Low and Medium Voltage (LV and MV) distribution networks, and 4) create a stable local economic framework less subject to wholesale price spikes. In this way, RECs are a promising alternative mechanism for the transition to a more flexible and sustainable energy system.

Renewable energy communities may give rise to strategic interactions between community members, who compete for common resources (e.g., the network, local production surplus, etc.), which are not captured by centralized models. In traditional optimization problems, decisions are made by a central operator, such as a community manager, which aims to optimize a single global objective function (e.g., the REC costs). In practice, community members may pursue different objectives, sometimes conflicting. Furthermore, they make decisions that primarily maximize their own interests. Another issue concerns the data privacy: members may be reluctant to share their personal consumption or production information at any time of the day. These information could provide their time of presence at home. Game theory, especially *noncooperative games*, offers a suitable framework for modeling these strategic interactions.

Game theory is a mathematical field that provides a set of analytical tools designed to model and analyze strategic interactions between decision-makers

[7]. It is based on the fundamental assumption that these decision-makers are rational [8], in the sense that they seek to optimize their individual objectives by considering the alternatives available to them, their expectations regarding uncertainties, and the expected behavior of the other participants. Thanks to their ability to model complex situations in an abstract way, game theory models offer a powerful framework for analyzing a variety of phenomena, from economic dynamics to social interactions and environmental challenges, considering the sometimes divergent objectives of the players involved. In this context, a game represents a framework defined in advance by rules, in which a set of individuals (players) must choose a strategy (action) to follow from a set of possible strategies. The situation resulting from the combination of decisions made by all the players, and in some cases by chance, is called a game outcome, and is associated with a payment or cost for each player. Game theory proposes solution concepts for classes of games, that systematically describe the rational behaviors of players and the resulting outcomes, while studying the properties of these solutions [7].

In this thesis, we focus exclusively on noncooperative games, where each player acts individually to optimize her own objective, without explicit coordination with other players. We study two groups of game-theoretic models: *strategic games* and *extensive games*. Strategic (or normal-form) games model a situation in which all players choose their strategies independently and simultaneously. Thus, each player chooses her action plan once and for all, and is not informed of the strategies chosen by the other players. These games are said to be static. The most popular solution concept for strategic games is the *Nash equilibrium*. On the other hand, extensive-form games offer a convenient approach to model sequential strategic interactions. These games specify the possible orders of events, so each player considers her plan of action not only at the start of the game, but also each time he makes a decision. For extensive games, we use the *subgame perfect equilibrium* as solution concept.

We would like to emphasize that the application of game theory in the energy sector is not a novelty in itself. Numerous studies have already demonstrated its usefulness for modeling and analyzing various strategic interactions in this field, see e.g., [9, 10, 11, 12, 13, 14, 15, 16, 17]. Therefore, this manuscript is part of an established academic continuity, while exploring specific aspects related to RECs and member behaviors.

1.2. Objectives and contributions

The main objective of this thesis is to model renewable energy communities and the various challenges surrounding them using noncooperative game theory. For that purpose, this work is divided into two parts, exploring a specific problem that can be modeled with a specific noncooperative game form. More particularly, this thesis aims to develop and provide extensive analysis of theoretical models that are flexible enough to be adapted to other configurations or parameters, rather than offering tools that can be directly used as such. When possible, our models are backed by numerical results.

The first part of the thesis explored the optimal day-ahead scheduling of energy exchanges and members' appliances inside renewable energy communities using *normal-form games*.

- We extend the existing literature on local market designs for energy communities by modeling the valuation of local excess generation internally, and we augment the grid cost structure by considering peak tariffs and testing an academic and a realistic tariff reflecting Belgian regulation. We formulated the mathematical problem in a centralized fashion (i.e., optimization-based), and distributed the REC total costs among community members ex-post using four allocation mechanisms. We also developed decentralized models based on noncooperative game theory, which endogenize cost distribution.

- We carried out an extensive theoretical and empirical study concerning the existence of equilibria with the decentralized models. We also study the efficiency of the obtained equilibria via the so-called *price of anarchy* (PoA) [18], i.e., we compare theoretically and empirically the total REC costs obtained at the worst equilibrium with the social optimum obtained with the centralized formulation. We first show that there always exists an equilibrium that is a social optimum. We also show that the computed equilibrium induces a total bill equal to or slightly different from the centralized solution, meaning that the faster optimization-based model can be preferred for macroscopic, system-level studies in which communities may be considered as single economic entities.

- We study and compare the members' individual outcomes for the centralized and decentralized formulations, for each cost distribution. We show empirically that replacing decentralization with ex-post allocation from the faster centralized model essentially keeps the same individual invoices for the three daily billing methods (non-negligible deviations occur for the continuous billing scheme with the academic grid tariffs), which is important information for community managers for billing purposes.

- We perform a sensitivity analysis of retail electricity prices and measure the impact on the total REC costs. We demonstrate the existence of a threshold in the import retail price, depending on the difference between the import/export community prices and the import/export retail prices, for which the economic gains of operating as a REC increase significantly, for both grid tariffs. Furthermore, we study the impact on members' individual bills and interest in joining/leaving the community. We show that, according to our hypotheses, the realistic grid tariff design is at least neutral or beneficial in terms of individual costs for each user type, provided certain cost allocation policies in place.

The second part of this work explores a fairly new topic: the integration of a new member into an existing renewable energy community. Indeed, European directives [6, 19] mandate that participation in an energy community be open and voluntary, adhering to transparent and non-discriminatory criteria. Likewise, members wishing to leave the community are entitled to a far and non-discriminatory exit process. However, the lack of detailed guidelines on these procedures creates ambiguities. This absence of standardized regulations introduces uncertainties and potential challenges for energy communities. The contributions made by this work to fill this gap can be listed as follows.

- We present an original approach of the new member integration problem into an existing REC, modeled using extensive games. The problem considers both long-term strategic decisions (investments, price adjustments) and short-term decisions (day-ahead schedules). The theoretical models developed, offer enough flexibility, and can be extended to encompass a variety of scenarios and stakeholder preference criteria (economic and environmental). In addition, prospect theory is used to model the bounded rationality of participants, more specifically on their perception of retail import prices, providing a better understanding of their behavior under uncertainty and risk.

- We applied our models to a detailed case study
  - An analysis was carried out on the results of heuristic methods from the literature [20]. Compared to the subgame perfect equilibria obtained when the community initiates integration, these metrics can effectively predict the selected profile when the REC has financial criteria, such as the net present value maximization or the total cost

minimization. However, their reliability decreases if the REC follows criteria such as the minimization of carbon emissions or the price per kWh.

– We conducted an extensive parametric study to demonstrate the flexibility of our modeling framework. Simulations have revealed that the outcomes at subgame perfect equilibria, and the behavior of stakeholders are influenced by various aspects of the problem: the order of decisions, preference criteria of the candidates and the REC, as well as the prospect theory. The order of decisions and stakeholders preference criteria thus modify the strategies adopted, which can lead to solutions that are more focused on community or individual objectives, sometimes to the detriment of the other participant. Furthermore, the integration of prospect theory shows that stakeholder choices introduce deviations from the behavior predicted by perfect rationality, thus impacting the final results. These deviations are mainly due to the parameters of the PT functions and, in particular, to the reference point selection method.

1.3. Thesis organization

The present thesis covers a wide range of interdisciplinary fields, such as the energy sector, mathematics, operational research and economic. Thus, we provided specific chapters dedicated to the contextualization and presentation of fundamental concepts used in this report. This manuscript is structured as follows.

- **Chapter 2** establishes a general background for the context in which this thesis is situated. It begins by describing the European electricity supply chain, with its various system actors and associated markets. It then introduces the energy communities and identifies the issues addressed in the next chapters.

The next two chapters constitute the part: "*Strategic Games for Day-ahead Scheduling*".

- **Chapter 3** provides the necessary mathematical theory and tools for modeling and solving the developed models in the first part of this thesis. Hence, some notions and results of convex optimization, game theory and variational inequality theory are covered.

- **Chapter 4** develop two market designs bases that dictate the energy exchanges inside RECs, taking into account the short-term energy resource planning. Two grid tariff structures are also proposed. Both designs are formulated as optimization problems first, and as noncooperative games then. The existence and efficiency of the decentralized equilibria are studied theoretically and empirically. In addition, a sensitivity analysis of retail electricity prices is applied.

The next two chapters constitute the part: "*Extensive Games for New Member Integration with Investment*".

- **Chapter 5** introduces the theoretical concepts of extensive games and prospect theory, offering powerful analytical tools for modeling sequential strategic behavior. It also presents the prospect theory, that can be used to describe actual human behavior in decision-making processes.

- **Chapter 6** explores the issues involved in integrating a new member and her investments into an existing energy community, with a particular focus on extensive game formulations. We analyze different preference criteria (economic and environmental), as well as the consideration of stakeholders' bounded rationality by means of prospect theory.

- **Chapter 7** summarizes the main contributions of the thesis and proposed some perspectives for future research.

The developments underpinning the research contribution in this Ph.D. thesis are related to mathematical convex optimization, game theory, variational inequality theory and prospect theory. We use Python [21] and Julia Programming Language [22] together with the modeling language JuMP. We use the Gurobi solver [23] to optimize the resulting models.

1.4. List of publications

The following publications reflect the research contributions incorporated in this thesis.

Chapter 4 is based on the three following papers:

- [24] L. Sadoine, M. Hupez, Z. De Grève and T. Brihaye, "Towards Decentralized Models for Day-Ahead Scheduling of Energy Resources in Renewable Energy Communities," in *Operations Research Proceedings 2022*, Springer International Publishing, 2023.

- [25] L. Sadoine, Z. De Grève and T. Brihaye, "Impact of retail electricity prices and grid tariff structure on the operation of resources scheduling in

Renewable Energy Communities," in *2023 IEEE PES Innovative Smart Grid Technologies Europe (ISGT EUROPE)*, 2023.

- [26] L. Sadoine, Z. De Grève and T. Brihaye, "Valuing the Electricity Produced Locally in Renewable Energy Communities through Noncooperative Resources Scheduling Games," under revision in *Applied Energy*.

Chapter 6 is based on the following paper, which is currently in preparation:

- [27] L. Sadoine, Z. De Grève and T. Brihaye, "New Member Integration Problem in Renewable Energy Communities: An Extensive Game Study with Prospect Theory," in preparation.

The following publications have been produced during the course of the PhD thesis. Although their content is relevant to the overall research context, it is not directly included in this manuscript:

- [28] J. Allard, A. Rosseel, L. Sadoine et al., "Technical impacts of the deployment of renewable energy communities on electricity distribution grids," *27th International Conference on Electricity Distribution (CIRED)*, 2023.

- [29] J. Allard, L. Sadoine, L. Liégeois, T. Brihaye, F. Vallée and Z. De Grève, "Rule-based optimization for energy communities demand-side scheduling and settlement," in preparation.

CHAPTER 2.

European Electricity System: Towards a Decentralized Prosumer-Centric System

This chapter provides an introduction to the European electric power system and the electricity supply chain, a complex interconnected grid linking electricity production and consumption centers that plays a crucial role in supplying energy to millions of people [30, 31]. More specifically, it introduces the basic physical characteristics of the European power grid, the dynamics of energy markets, which aim at coordinating production and consumption activities, and the emerging role of prosumers, i.e., consumers who produce and consume their own energy. This presentation enables a better understanding of the factors shaping the European power system and the need for the system to adapt to face growing societal and environmental challenges.

The energy transition to a decarbonized, reliable and sustainable energy system has become a priority. The massive integration of renewable energy sources (e.g., wind or solar) and electrification of end use, rapid technological development, and growing climate change concerns are key factors in the energy transition,

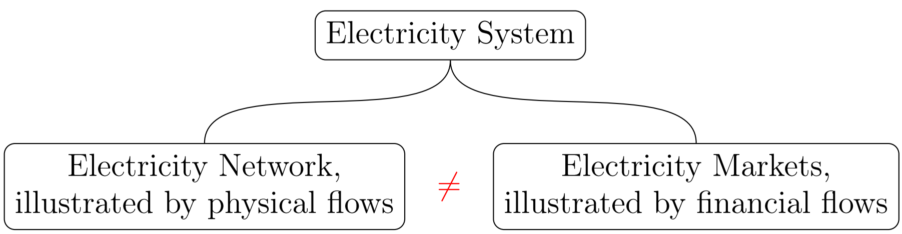


Figure 2.1.: The electrical system with the electricity grid illustrated by physical energy flows and electricity markets illustrated by virtual financial flows.

and are profoundly transforming the electricity sector. Indeed, one of the main technical challenges in electricity systems consists of ensuring instantaneous balance between production and consumption, so as to ensure frequency stability and avoid cascade disconnections, which may ultimately lead to a blackout. Large-scale storage could help in that respect, but is, however, currently beyond reach, and is limited to specific landscape configurations in the case of the pumped storage hydro technology (which is also often in conflict with other land uses). In that context, it becomes essential to adopt the short-term operational management procedures (e.g., day-ahead scheduling of production and consumption assets, network reconfiguration, etc.) in electric power systems to cope with the limited predictability and variability induced by weather-dependent renewable energy sources, thereby ensuring the instantaneous system balance and frequency stability. Flexibility, i.e., the ability of production and consumption to adapt their energy output on short notice following a signal sent by e.g., the system or market operator, will play a greater role in ensuring that balance.

Challenges are not only technical but also relate to the economics of electricity. Indeed, the liberalization of the electricity supply chain, which occurred in the late 1990s, is now showing its limits more than ever: the events of the last few years, such as the COVID-19 pandemic and the geopolitical situation in Eastern Europe, have thrown the gas and electricity European markets into an unprecedented crisis [1], with markets failing to prevent extra profits from some producers, highlighting the necessity of new measures to protect end-users. Although able to ensure coordination between production and consumption at the European scale, liberalized electricity markets are furthermore failing to provide stable price signals that stimulate sufficient investment in new decarbonized generation assets, thereby calling for new market mechanisms, such as capacity markets, whose efficiency is still open to debate [32, Ch 5].

In parallel, the growing share of decentralized production (e.g., photovoltaic (PV) panels, wind turbines), affordable Energy Storage System (ESS) and flexible systems like Electric Vehicles (EV) in the residential and industrial sectors, is underlining the population's greater sense of responsibility, who is willing to play a more active role in the energy supply chain. These Distributed Energy Resources (DERs) are usually connected to electricity distribution networks, and call for new consumer-centric market mechanisms. These mechanisms must be able to leverage the flexibility available on the end-user side through increased coordination (e.g., Demand-Side Management (DSM) programs), while partially protecting consumers against energy crises and creating a favorable context for investment in local generation and flexibility assets. Among them, Energy Communities, as formalized by the EU Commission in its Directive

2018/2001 [6], appear to be a promising alternative, which is further developed in this thesis.

In the remainder of this chapter, we assume that the electricity system is European, unless explicitly stated otherwise. Section 2.1 discusses the liberalization process of the electricity system. Section 2.2 provides an overview of the physical infrastructure of the electricity system , the physics governing energy flows, and an introduction to the various actors involved and their role in the system. An overview of the market structures and mechanisms in the European electricity system is presented in Section 2.3. Section 2.4 discusses some of the current challenges faced by the system. Then, energy communities are defined, and the challenges associated with their modeling and implementation are discussed in Section 2.5. This last section also establishes the scope of this thesis.

2.1. Liberalization of the electricity system

The liberalization of the electricity system in Europe has been driven by various political, economic and technical factors. It was also part of a worldwide trend towards deregulation of previously regulated sectors (e.g., telecommunications, transport or banking) in the 1990s.

The components of the electricity system are grouped together in the electricity supply chain. This can be disentangled in the following five functions: production or generation, transmission, distribution, metering and retailing, and coordination (namely short-term scheduling of assets while ensuring a safe operation of the system, long-term planning to anticipate future demand trajectories, etc.). Since the 1920s, and more particularly after World War II, the electricity system in many European countries has been organized as a vertically integrated monopolistic structure where production, transmission, distribution, electricity retail and coordination were ensured by the same entity on a given area, as illustrated in Figure 2.2. End-users could only purchase electricity from this national or regional operator, at prices controlled by the State or the public utility. In order to break down such a monopoly, the European Union (EU) decided to deregulate electricity markets to create an unbundled structure open to competition.

In practice, the production and retail sectors are now open to competition, whereas network operations (transmission and distribution, depending on the voltage level) remain regulated natural monopolies (see Figure 2.3). A natural monopoly corresponds to a situation in which the most efficient way to organize

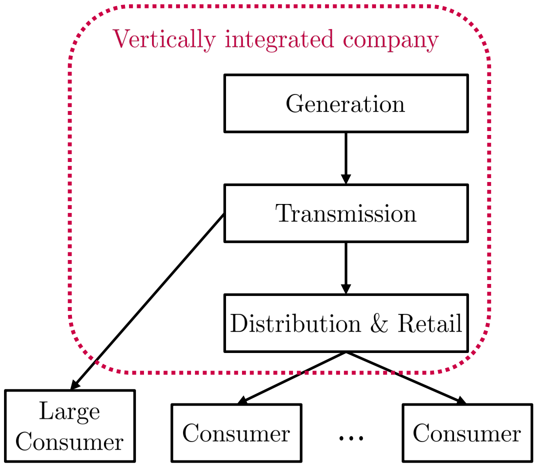


Figure 2.2.: The electricity system before the liberalization.

an activity is to entrust it to a single company, which is often the case in network industries (railway, etc.). In the present case, the infrastructure of transmission and distribution networks are so capital-intensive that a competition based on the multiplication (and even simple duplication) of physical infrastructure could only lead to destructive competition [32].

The overall coordination activity is therefore now shared between natural monopolies, such as network operators, which do not own production and consumption assets, and private actors, such as producers, retailers, and more and more consumers/prosumers. Markets have been introduced at two levels to ensure coordination between production and consumption: at the production level first (i.e., wholesale markets with many different time maturities, in which generators, large consumers and suppliers/retailers interact), and at the retail level then (i.e., retail markets, in which retailers and small to medium consumers and prosumers interact). Close to real time, the network operator remains responsible for the safe physical operation of the overall system, and rapidly adjusts the production and consumption levels of actors interacting through markets of "last resort" (e.g., reserve and imbalance settlement) under its supervision, in order to ensure system balance, voltage stability, etc. These market structures will be further described later in the present chapter.

Opening up production and retail activities to competition aimed to stimulate innovation, and improve the efficiency of electricity production and distribution, thereby reducing costs for end-users. However, after approximately 30 years of liberalization, the materialization of these benefits is still open to debate. Another consequence of market competition is the ease with which new electricity

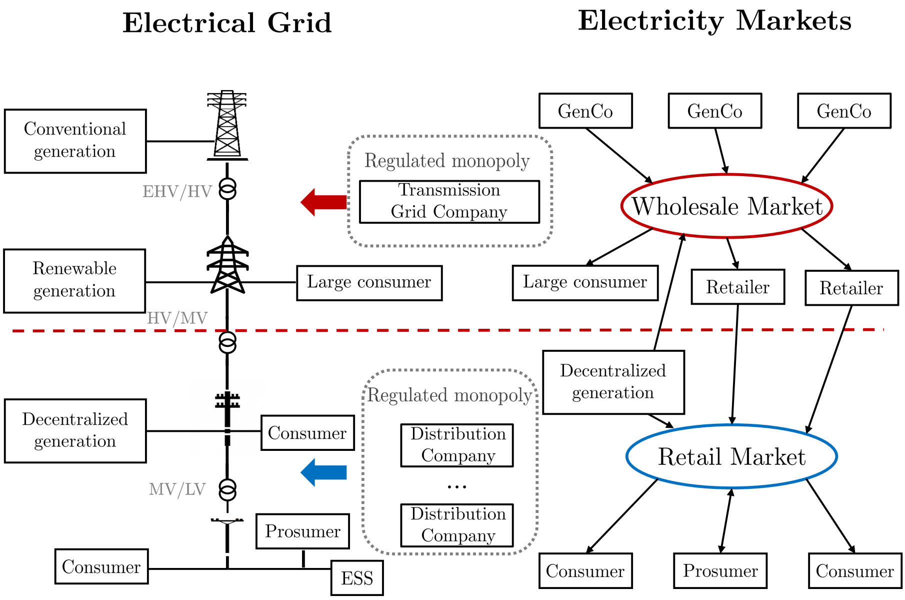


Figure 2.3.: Schematic representation of the current electricity system, consisting of the physical network (to the left) and a simplified vision of electricity markets limited to markets of energy, disregarding reserve, capacity markets, etc. (to the right).

producers can be introduced. One of Europe's objectives was to enable greater diversification of energy sources and thus encourage investment, particularly in renewable energy installations. Again, the right investments needed for the energy transition are slow, and the ability of markets to trigger the needed changes poses questions [33]. Furthermore, new actors have emerged, increasing complexity and redefining the framework of the electricity system. Their roles are described in Section 2.2.

National monopolies were largely closed to foreign operators, leading to a fragmentation of Europe's electricity markets. Each country operated almost in isolation with little interconnection between power grids; thus some countries could be more vulnerable to energy shortages or price fluctuations. The European Commission wanted to harmonize and integrate national electricity markets by creating common guidelines and rules. This enables better coordination between member countries, to promote fair competition and protect final users while guaranteeing a stable and secure energy supply. The ACER and ENTSO-E agencies were created, which correspond to the aggregation of, respectively, European regulators and electricity transmission system operators.

In 2019, the European Commission delivered the "Clean Energy for all Europeans" package, which provides a crucial legislative framework for the European Union to achieve its climate objectives for energy transition and carbon emission reduction, while pursuing the integration of competitive and sustainable energy markets [34]. This package also places a strong emphasis on the rights of electricity end-users. It introduces measures to enable them to participate actively in the electricity supply chain, notably through dynamic contracts, self-generation (e.g., PV, small wind turbines) of electricity and flexible management of their consumption (e.g., via smart metering technologies). It also introduced the concept of energy communities in its legislation: we refer to Section 2.5 for more details of this new mechanism. Consumers can choose their supplier more easily and are better protected against abusive market behaviors. Another package was released in 2021 with the aim of aligning the EU's energy targets with the new climate ambition for 2030 and 2050.

In short, the liberalization of electricity markets in Europe was aimed at improving competitiveness, efficiency and market integration, while supporting the energy transition and guaranteeing lower prices for consumers. Although the materialization of these benefits raises questions is still open to debate, we are currently undergoing a reform of the electricity system.

2.2. System structure and participants

The current electricity system is composed of physical infrastructure and of organized electricity markets. In Europe, electricity flows under the form of Alternate Current (AC) energy through a complex physical infrastructure, wherein a wide range of components (e.g., alternators or generators, transformers, protection devices, physical transmission and distribution lines, power electronics converters such as inverters, decentralized generation, and industrial and domestic consumption) are interconnected across a wide geographical area. The physical infrastructure is commonly subdivided into two parts 1) the transmission system and 2) distribution systems. Most electricity is produced by large power plants connected to the transmission grid. The transmission network is meshed and is composed of the high and very high-voltage (HV) three-phase lines (400kV-30kV in Belgium), which carry the electricity generated over long distances at national and international levels, and then feeds geographical zones through distribution grids. Note that large industries can directly be connected to the transmission grid (see Figure 2.4). Distribution grids are radial and include the medium (MV) and low-voltage (LV) lines (from 30kV to 230V) which supply industrial and residential consumers.

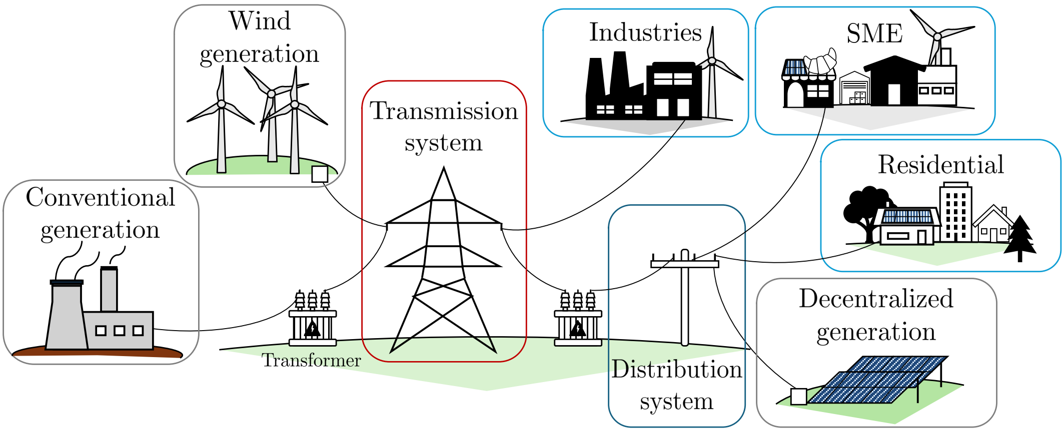


Figure 2.4.: Simplified representation of the electrical grid.

The AC power system relies on a constant balance between production and consumption, as any difference can affect grid stability. This is because electricity is one of the few energies that cannot be stored efficiently at a large scale and for long periods of time. Current solutions, such as batteries or pumped-storage stations (whose deployment is restricted by the land and relief requirements for installation), are still very expensive and limited in capacity, and raise their own environmental concerns. Any imbalance between offer and demand affects the grid frequency, which must remain stable (the nominal frequency is set at 50 Hz in Europe). A significant deviation in frequency can lead to malfunctions of electrical appliances and cascade disconnections, which may ultimately lead to the collapse of the network (or blackout). As a result, electricity must be consumed immediately after it is produced, and a constant balance must be maintained between production and consumption. This balance is monitored and maintained physically by the *transmission system operator* and economically by *balancing responsible parties*, as described in the next sections. The increasing share of renewable generation, such as wind and solar power, which depend on the weather, makes production more intermittent and difficult to predict, thus complicating the balancing task.

Electricity flows cannot be guided easily, although modern power electronics converter systems, such as Flexible Alternating Current Transmission Systems or FACTS, provide new, but still expensive, possibilities for controlling electricity flows in high-voltage AC lines. Electricity follows naturally the path of least impedance through Kirchhoff's laws, which may cause technical issues such as over/undervoltages or congestion when production and consumption are dispatched though markets that do not endogenize all the grid constraints. In that case, the transmission network operator may handle the issues with its own assets (e.g., on-load tap changers, etc.), but if not sufficient, may contract with market actors to get the needed support, though ancillary services markets.

2.2.1. Producers

In liberalized electricity markets, producers are private companies. They generate electricity by operating and owning large power plants or renewable energy source farms, which are usually directly connected to the transmission grid. Energy producers aim to maximize their profits by selling their generations to the wholesale market, interacting with balance responsible parties. Note that producers can also act as electricity retailers: we speak of vertical integration of production and retail activity. Different production costs are possible, depending on the level of technical complexity of energy generation. Some examples of producers in Belgium: ENGIE (Electrabel), Luminus (EDF group) or Eneco which focuses exclusively on solar energy, wind power and biomass.

2.2.2. Transmission system operator

The transmission grid is managed by the Transmission System Operator (TSO). It owns, maintains, operates and builds the physical infrastructure of the transmission network in order to integrate the development of new generation facilities and interconnect the neighboring countries. It acts as a market facilitator and ensures that all customers have equal (non-discriminatory) access to all resources necessary for their trade. In exchange for being granted a regional monopoly, the TSO must accept that the regulatory authorities will determine its revenues by fixing for instance transmission grid tariffs paid by end-users. The generation and consumption balance is ensured in last resort by the TSO, which is legally prohibited from owning power generation units. So, it is not able to ensure directly by itself the stability of its network and operates dedicated markets (reserve and imbalance settlement, see later), but it can monitor and coordinate the different actors of the system while preventing voltage violations and line congestion. In Belgium, the company responsible of the transmission system is Elia. The organization managing the coordination among the different national TSOs is the European Network of Transmission System Operators (ENTSO-E).

2.2.3. Distribution system operators

The safe operation and planning of the electricity distribution system is managed by Distribution System Operators (DSOs). They own, maintain and invest in the distribution grid infrastructure, which connects most final end-users (households, small and medium-sized enterprises (SME), and public infrastructure). They install electricity meters and communicate the metering to the suppliers. Each DSO has a regulated monopoly on a defined area. End-users are therefore dependent on a DSO based on the location of their point of connection, and cannot choose their DSO. Similarly to the transmission system, the costs associated with the management of the network, known as grid fees, are passed on to the final end-users via regulated tariffs. Some regional DSOs in Belgium: ORES (Wallonia), Fluvius (Flanders) or Sibelga (Brussels).

The roles and missions of distribution system operators (DSOs) are currently evolving in the presence of increasing decentralized generation (e.g., PV farms, biomass) and the emergence of prosumers. They now have to manage a network where electricity flows are bidirectional: not only from large producers to consumers, but also from small local producers to the upstream grid. This means modernizing the distribution network to absorb these new local resources, and implementing intelligent solutions to ensure network flexibility and stability. They also have to adapt to an increasing electrification of end uses (e.g., mobility, heat/cold), in line with the decarbonation of society. They play a crucial role in the energy transition, facilitating the integration of renewable energies and demand-side management systems.

2.2.4. End-users

End users are energy consumers of various sizes, spread throughout the system. These include residential households (connected to the LV network and buying energy on the retail market), small and medium-sized enterprises (SME) or the tertiary sector (connected to the LV or MV network and buying energy on the retail market), as well as large industrial actors (connected to the HV network, which are able to participate directly in the wholesale market). They obtain their energy from the main grid by contracting a supplier of their choice.

In recent years, with the development of decentralized production technologies, the role of end-users in the electricity system has evolved with the emergence of *prosumers*. A prosumer is an end-user who is both a consumer and a producer of electricity. Prosumers produce part of their electricity locally using personal generation installations, such as PV panels, mini wind turbines or cogeneration systems. If their production exceeds their electricity needs, prosumers can inject this surplus into the main grid and/or store part of it using domestic storage technology. On the other hand, if their production does not meet their consumption, they extract energy from the grid. This new role changes the dynamics of the electricity market, making users more active and independent of centralized producers. It also has an impact on power grid management, particularly in terms of flexibility and the balance between production and consumption.

2.2.5. Retailers

Introduced during the liberalization, retailers are intermediate actors who make the link between the wholesale and retail markets. Retailers or electricity suppliers are companies that either own generation means and/or buy electricity from the energy markets, and sell this energy to end-users. Suppliers compete in the retail market on their electricity prices, which is considered a commodity. End-users can freely choose their supplier based on the different pricing plan offers. Suppliers can offer several types of contract to set themselves apart from the competition; they can propose different durations (one or more years), attractive prices or tariff formulas (fixed tariffs or based on real-time market fluctuations), or provide electricity exclusively from renewable sources. They also propose specific commercial contracts for prosumers, where the retailer can buy the electricity surplus injected by the prosumers.

Retailers are also responsible for the billing, which compensates the other implied parties (TSO, DSO, public authorities, etc.). This billing differentiates into different components: energy commodity (subject to competition), grid fees as well as taxes and levies (regulated).

2.2.6. Balancing responsible parties

Balance Responsible Parties (BRPs) are entities appointed by the TSO to help maintain the balance between generation and consumption on the network. It can be either a producer, an important industrial company, a retailer or a trader. Each BRP must forecast and take all reasonable measures to preserve the balance between injections, off-takes and commercial exchanges within a portfolio of one or more access points. Thus, it has the responsibility of composing a daily balancing schedule of its portfolio on a quarter-hourly basis. BRPs are financially responsible for the potential imbalance. Producers and large consumers can ensure this role, while residential end-users are usually represented by suppliers, which act as a BRP.

2.2.7. Aggregators or flexibility service providers

An aggregator or a flexibility service provider is a third party that combines and manages the flexibility of several consumers, prosumers and small decentralized generation units. Electrical flexibility is the ability to voluntarily increase or decrease production or consumption compared to normal usage, in response to external signals or local measures. Such portfolios have a significant volume, capable of intervening in the markets. Aggregators provide balancing services to the grid operator to compensate for the imbalances in the power grid during consumption peaks (e.g., cold waves) or excess energy on the grid (too much solar or wind power) by adjusting generation and consumption, or may provide other types of flexibility services linked to voltage and congestion management, etc.

2.2.8. Regulators

The liberalization introduced independent organizations called regulators to monitor both regulated and market-related activities (which does not mean that the system was not regulated nor monitored during the monopolistic era). Regulators are entrusted to ensure transparency and competitiveness in electricity markets, with the driving goal of serving the public interest. They aim to protect end-users by monitoring energy prices and illegal market behaviors. They also ensure that the regional, national and European regulations are correctly integrated into the market operations. The federal Belgian organism in charge of the regulation of the Belgian transmission system, of nuclear generation, of offshore wind generation, etc. is the CREG (Commission of Regulation of Electricity and Gas). The regional regulators (VREG, CWaPE and Brugel) have local responsibilities pertaining to regions: distribution network tariffs, renewable energy subsidies, new regulations on energy communities, etc. The ACER is the entity gathering regulators at the European level.

Figure 2.5 provides an overview of the main interactions between the actors and the main electricity markets (see Section 2.3).

2.3. Electricity markets

In contrast to conventional financial markets, where commodities or assets exchanged are often intangible or easily stocked, the characteristics of electricity strongly influence the way in which this commodity is traded. Demand varies greatly throughout the year, but electricity cannot be economically (and physically) stored on a large scale and must be produced, transported and consumed in real time. It is also necessary to balance production and consumption to maintain system security and stability. Another distinction lies in the separation between economic and physical flows. In electricity markets, financial exchanges are virtual. They take the form of contracts or transactions

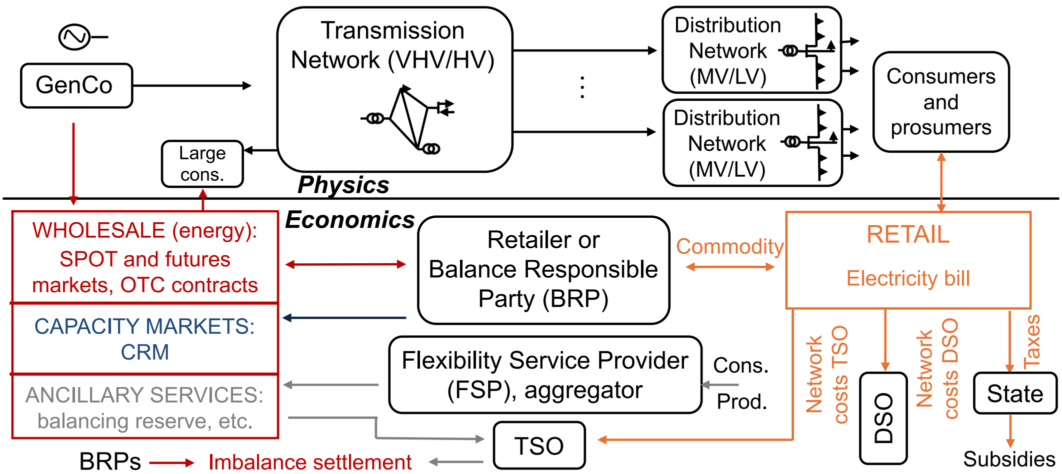


Figure 2.5.: The main actors and their interactions within the electricity markets.

between buyers and sellers, which establish the price and quantity exchanged. These economic flows do not, however, directly influence the path taken by electricity, which obeys complex physical lows.

These specific constraints make electricity markets fundamentally different from other commodity or financial markets. Therefore, there are various markets to answer different needs at different time horizons and ensure a proper operation of the electricity supply chain. These markets are structured around three fundamental elements: the nature of the product traded, the time of the trade and the place of delivery. The general structure of electricity markets can be summarized as in Figure 2.6.

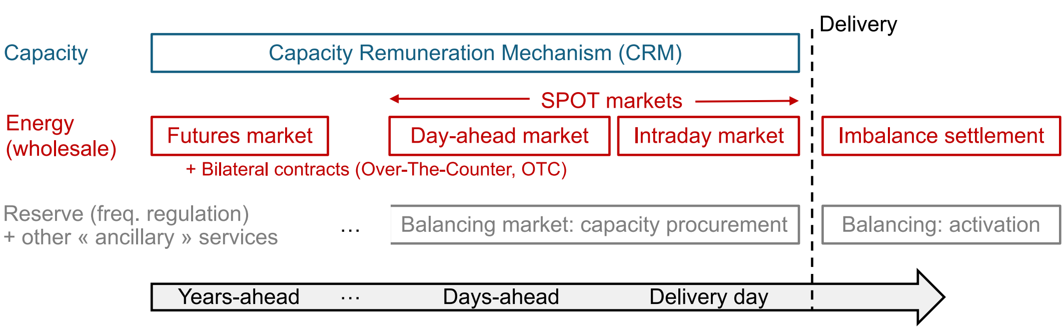


Figure 2.6.: General structure of electricity markets.

2.3.1. Wholesale electricity markets

The *wholesale electricity markets* are places where electricity is bought and sold in large quantities, generally between producers, suppliers and large consumers (e.g., industries). Two types of electrical energy exchange coexist in Europe: 1) *over the counter* (OTC) markets and *power exchange* (PX) markets. In OTC markets, the participants negotiate one with another (bilateral contract) without a central physical location. The main advantage of such contracts is that they can be completely customized to fit a customer's requirements on electricity volume and prices, time horizons, without others knowing the details of the transaction. Such exchanges have little transparency and are less regulated. PX markets offer full electronic platforms for multilateral and anonymous transactions. Markets are differentiated according to their time horizon.

Long-term markets

The *long-term markets* enable participants to exchange electricity for future delivery (from day-ahead up to a few years). These contracts guarantee a price and quantity well in advance, limiting exposure to price fluctuations on the spot markets. These are referred to as forward contracts on OTC markets and futures contracts on power exchange markets.

Day-ahead markets

In *the Day-Ahead (DA) markets*, electricity is exchanged for next-day delivery (D-1). Each day, BRPs submit their offers or bids on the platform for each hour of the following day. For each time step of the horizon considered, prices and volumes are cleared based on a merit order mechanism. The market price (π^*) and volume (Q^*) (or the *market equilibrium*) are provided by the intersection of the supply and demand curves as represented in Figure 2.7. The supply curve represents the available electricity quantities that producers are willing to sell at their minimum prices. The higher the price, the greater the supply. Meanwhile, the demand curve shows the quantities demanded by consumers at the maximum price they are willing to pay. The lower the price, the higher the demand.

The market clearing process matches the offers and bids to obtain the market prices and volumes for each hour of the next day, which corresponds to maximizing the social welfare of the market under perfect competition hypotheses. Social welfare is equal to the difference between total utility demands and the

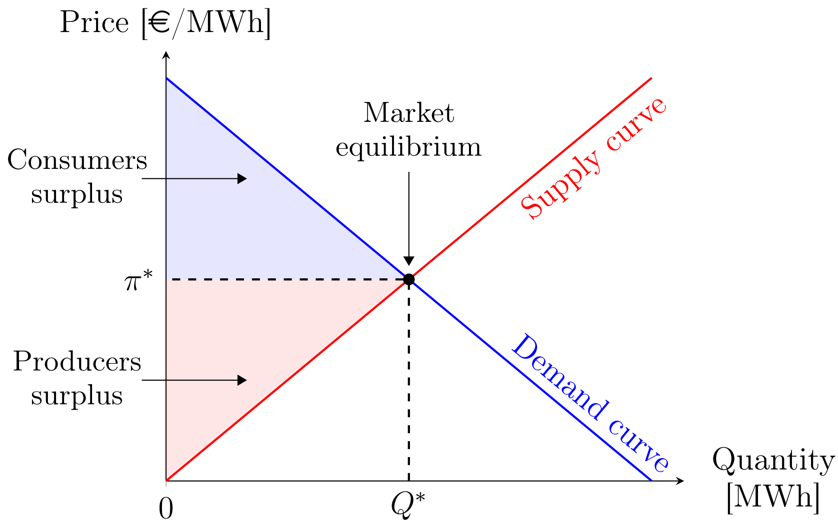


Figure 2.7.: Market Equilibrium: the clearing price π^* and volume Q^* are set by the intersection between demand and supply curves.

total cost of generators. It can also be seen as the sum of consumers' and producers' surpluses.

Based on these prices, the BRPs will self-schedule the generation and flexible consumption assets within their portfolio, so that the day-ahead spot market, beyond setting prices for the upcoming day, is implicitly responsible for the effective dispatch of generating units for the next day.

Prices can be influenced by several factors, such as demand forecasts, variable power plant costs and weather, which affects renewable production. Furthermore, if renewable generation is abundant, its very low marginal costs tend to drive down prices on the DA markets. On the other, when production is limited, prices can rise quite a lot. The Belgian market operator is EPEX SPOT (or Belpex). The day-ahead contracts can also be negotiated through OTC markets.

Intraday markets

The *intraday markets* are continuous markets that enable electricity exchanges for same-day delivery, with transactions closing up to five minutes before actual delivery, in order for BRPs to balance their portfolio closer to real time. These typically involve organized OTC transactions that are regularly settled through PXs. These markets are particularly important for managing potential discrepancies between forecasts made in day-ahead and actual conditions (e.g., weather forecasts not as accurate as expected), or for reacting to sudden unforeseen changes, such as power plant outages. BRPs can thus intervene quickly to adjust their transactions to balance their portfolios.

2.3.2. Ancillary services and balancing markets

Since the liberalization of the electricity market, transmission system operators (such as Elia in Belgium) no longer own the generation assets they need to ensure the security of electrical system operations. Consequently, they procure the services needed to maintain grid stability and balance via contracts with specific providers. These *ancillary* or *system services* include mechanisms such as frequency and voltage regulation, the provision of power reserves, and black-start services.

We focus here, more particularly on balancing services settled through reserve markets. Although BRPs take every precaution when participating in the spot markets (i.e., day-ahead and intraday) to achieve the best possible balance in their portfolio, real-time imbalances may remain. Real-time imbalances are corrected by the TSO using balancing products purchased on the *balancing (or reserve) markets* so as to ensure frequency stability. A range of products with different response speeds are available (Figure 2.8):

- The Frequency Containment Reserve (FCR), or primary reserve, should be active within the 30s to stabilize the network frequency in the case of an imbalance between production and consumption. It is automatically activated in response to a real-time frequency deviation.
- The automatic Frequency Restoration Reserve (aFRR), or secondary reserve, corresponds to a type of reserve that must react in 5 minutes in order to gradually restore the frequency to its constant level of 50 Hz after the FCR response.
- The manual Frequency Restoration Reserve (mFRR), or tertiary reserve should be activated by the TSO when facing a large and persistent imbalance. This reserve typically engaged within 7 to 15 minutes.

Usually, reserve markets entail two payments to the service provider: a payment for capacity procurement (whether it is activated or not), as well as a payment for the effective activation of balancing resources.

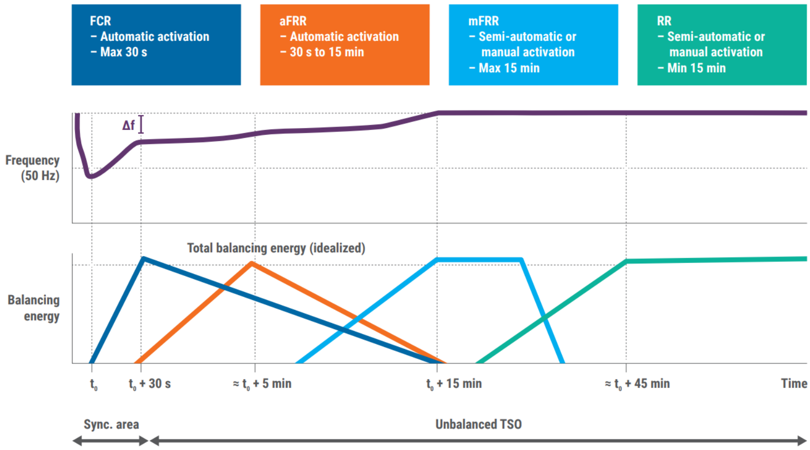


Figure 2.8.: Frequency restoration process. Source [35].

2.3.3. Imbalance settlement

The imbalance settlement is a real-time electricity market, in which BRPs are financially penalized or rewarded. BRPs are penalized for their individual imbalances that which worsen the overall balance of the electrical system, whereas those which helped maintain system balance are rewarded. The cost or gain received for each BRP is directly linked to the costs faced by the TSO for procuring and activating balancing reserves (see [36]).

2.3.4. Capacity markets

The price signals sent by long-term and spot markets are not sufficient to trigger investment in the generation assets needed to ensure coverage of demand, which are particularly CAPEX-intensive. This is partly due to the extreme volatility of electricity prices on spot markets, which do not reassure private investors who seek stable profits and high returns on investment.

The second main explanatory factor is the so-called *missing money* problem. Indeed, under perfect competition and additional (strong) hypotheses, it can be shown that profits made by inframarginal producing units (i.e., units whose marginal production costs are below the market equilibrium price) are just sufficient for producers to compensate for their fixed costs (i.e., investment plus operation and maintenance costs), and ensure the renewal of the electricity generation mix. This is verified provided that the market price is authorized to take extremely high values (usually assumed to be equal to the Value of

Lost Load or VOLL, i.e., the value attached by large consumers to shed their scheduled demand) for a few hours a year, corresponding to peak demand situations. In practice, regulators and public authorities are reluctant to let wholesale prices reach values of tens of thousands of euros per MWh, so that they tend to impose caps on market prices (4000€/MWh in Belgium currently). This leads to a situation where power plants, and especially peak units, which are in-the-merit only a few hours a year, do not recover their fixed costs, leading to the missing money problem.

Capacity markets have been introduced to mitigate that phenomenon, i.e., to provide an additional stable source of revenue for actors so as to trigger investment in generation assets. In Belgium, Capacity Remuneration Mechanisms (CRM), which are based on an auction system, have been introduced in 2021 (more information on [37]). Scarcity pricing, which can coexist with capacity markets, is another option investigated in [38].

2.3.5. Market coupling

We review briefly here the market coupling mechanisms in application in Europe. The EU is divided into different market zones, corresponding to countries (or regions depending on the case). The Belgian market zone is implicitly coupled with other European market zones through a mechanism known as market coupling.

The principle in day-ahead markets is to match the highest purchase bids with the lowest sales offers, regardless of the bidding zone in which they have been introduced (i.e., demand in Belgium 'sees' cheap production in Spain, which would not have been possible without coupling), while accounting for the available cross-border transmission capacities through the so-called Flow-Based Market coupling. The EU day-ahead market is currently cleared with a single complex optimization algorithm called EUPHEMIA. This results in clearing prices for each market zone, which are identical in the absence of cross-border congestion and may differ in case of congestion. The energy and interconnection capacities are thus traded together.

2.3.6. Retail electricity markets

The *retail markets* are supposed to allow consumers and prosumers to benefit from advantageous prices thanks to a certain number of competing suppliers. With deregulation, customers can now choose their supplier contract, which varies according to the commodity rate (fixed, variable or dynamic) proposed over a period of time, the energy origin and the services offered.

In fact, the electricity price *per se* is the only aspect of the energy bill that is subject to competition. The rest of the costs are regulated and uniform regardless of the supplier chosen. Under normal conditions (i.e., no energy crisis like the Ukrainian crisis), around 50% to 70% of the final bill comes from grid fees set by the TSO and the DSO, and various taxes and contributions imposed by the state (e.g., taxes to support renewable energies).

The current electricity bill for a domestic user in the Belgian framework is composed of several components:

1. Commodity costs. The energy cost depends on the end-user's consumption and the commodity price defined by the supplier. This price represents the electron price and is subject to competition.

2. Distribution costs. These costs may be divided into three portions, depending on the country and region: 1) the energy (or volumetric) part, which is proportional to the kWh consumed by the end-user over a given period; 2) the power (or capacity) part, which depends on the peak consumption of the consumer over a given period (e.g., the last 12 months); and 3) fixed costs depending on the max contracted power at the point of connection, including the metering and other management fees of the distribution network by the DSO.

3. Transmission costs. The transmission costs pertain to the electricity transmission grid and are regulated, and may follow the same structure than the distribution tariffs, although with different unit prices.

4. VAT and taxes. These costs are collected by public authorities and include energy policy enforcement, subsidies to renewable energies, public lighting, and other taxes.

All these costs are collected by the supplier or retailer, and grid components are passed to TSOs and DSOs, whereas taxes are passed to the public authority. The DSO is in charge of the metering of electricity.

An estimation of the electricity bill components of the Belgian invoice for a Walloon residential consumer in August 2024 is provided by CREG [39].

Energy Communities, which will be detailed later on, consist of new market mechanisms that occur at the retail level.

2.4. Some current challenges for the electrical system

The electrical system is undergoing a profound transformation as it faces challenges posed by climate change, technological advances and evolving regulatory frameworks. This section displays two key dimensions of these challenges: the decarbonization of society and the changing roles of actors within the energy system.

2.4.1. Decarbonization of society

The events of the last few years (the COVID-19 pandemic and the geographical situation in Eastern Europe in early 2022) have significantly stressed natural gas markets in the EU, which has in turn driven wholesale electricity prices to unprecedented peaks and volatility in most EU countries [1]. In addition, the global demand for energy is becoming even more pressing, meaning therefore that Europe is facing considerable energy pressure. Europe's energy supply still relies heavily on imported fossil fuels. What's more, some countries have been relying heavily on Russian supplies. The need to make rapid progress towards decarbonization has taken on an entirely new dimension, namely that of energy security and independence. In response to the energy crisis, the EU launched the REPowerEU plan in 2022 to reduce its dependence on Russian fossil fuels and accelerate again the transition to renewable energies. The solutions are similar to those proposed to address climate change: promote renewable energies, encourage innovation in new energy sources and vectors and optimize energy use, minimizing consumption through gains in energy efficiency and/or better energy sufficiency.

Challenges emerging from the production side

Massive integration of renewable energies, such as wind and solar, brings some challenges due to their intermittent nature and decentralization. The intrinsic uncertain nature of wind and sun raises the issue of availability. The wind speed and the solar radiance fluctuate along with the meteorological events, whereas classic power plants are fully or partially controllable. This drop in controllability creates a greater need for flexibility in the power system to maintain the balance between production and consumption at any time. Several solutions are emerging to deal with this intermittency at different levels.

Intermittency of renewable generation. The intermittency of renewable generation can be counteracted by leveraging flexibility on the demand-side at a local level. Consumer flexibility means adjusting their loads according to production availability, rather than just modulating generation to meet demand. It is based on several mechanisms and programs; these latter are referred to as Demand-Side Management (DSM) [40], which enable electricity consumers' demand to be adapted in real times by giving them particular incentives. These techniques encourage consumers to reduce or shift their consumption during periods of high demand or low renewable production. They receive financial compensation or benefit from more advantageous tariffs in exchange. End-users can also modulate their loads according to market signals via dynamic tariffs. As mentioned in the subsection 2.2.7, aggregators pool the flexibility capacities of several end-users to create a significant flexibility offer on the electricity markets. An appropriate regulatory framework is needed to ensure fair remuneration for services rendered to the power system and there develop consumer flexibility to its full potential. Flexibility consumption has the potential to flatten consumption peaks and manage RES variability. It is an important lever for reducing capacity needs.

Another solution is to develop storage technologies connected to the power grid to smooth out renewable production fluctuations. Electricity can be stored by electrochemical means or by coupling electricity with other energy vectors. Energy storage provides flexibility to balance supply and demand on the grid, by storing excess electricity produced during periods of high generation and releasing it during periods of low production. They are deployed in a centralized way, notably through large installations coupled with PV or wind turbine parks. Pumped-storage hydropower is the most widely used. At the same time, there is growing development of domestic or industrial storage systems, mainly via batteries with the lithium-ion battery currently the most common in these applications. Although the cost of batteries has fallen considerably in recent years thanks to the increasing production of electrical vehicles (EV), market disruption and competition between manufacturers of these vehicles have led to higher prices for the key minerals used in battery fabrication, notably lithium. Therefore, further cost reduction depends not only on technological innovations, but also on the evolution of the prices of these minerals [41].

Decentralization of generation. The increase in small-scale production units, such as PV panels on households and local wind turbines, is revolutionizing grid management. Once centralized, the system is becoming a decentralized paradigm more complex to manage at the distribution grid level. The distribution networks were indeed originally designed for the transit of unidirectional power flows from the transmission grid to end-users. They now interconnect an ever-growing number of decentralized production units, storage units, and consumers and prosumers increasingly able to modulate their consumption based on signals and market prices. This development goes along with an ever-increasing need for coordinated active network management of these units to handle bidirectional flows, guaranteed grid service quality and optimized use of generation and storage resources. Indeed, the distribution grid has to manage local injections of electricity, which can lead to congestion or power surges if left unchecked. Furthermore, if not effectively monitored, this can lead to "PV tripping", i.e., the forced reduction of solar production when the grid can no longer absorb the energy surplus. Active network management helps limit these losses by integrating solutions such as DSM, local storage and microgrids. This requires investment in digitization and real-time network monitoring.

Given the low energy density of renewable energy sources, such facilities usually have much smaller power capacities and require more space, but they have the advantage of being able to be installed closer to the load they serve through the distribution grid. Energy density refers to the amount of energy produced per unit area. This leads to offshore developments and can pose challenges in terms of cohabitation with other land uses (fishing, shipping). For instance, in the Ostend declaration, a coalition of nine countries (including Belgium) pledged to turn the North Seas into a Green Power Plant in Europe, and together aim for at least 120 GW of offshore wind power by 2030, and over 300 GW by 2050. Land-based projects can meet with local resistance. Neighbors may express concerns about landscape degradation, noise pollution, declining property values and impact on local biodiversity.

Challenges emerging from the consumption side

Aside from local renewable production sources, other DERs (such as distributed storage), installation of smart meters and associated communication technologies have emerged in modern households. As technology advances, there is a trend towards replacing technologies or processes using carbon-intensive fossil fuels with electrically-powered equivalents. We are witnessing the *electrification* of the loads in several sectors, such as

- **Heat sector** through massive deployment of domestic *heat pumps* (HPs). However, HPs provide only around 10% of the world's building heating needs. If we want to achieve carbon neutrality by 2050, the global heat pump stock will have to almost triple by 2030 to cover at least 20% of heating needs. This will require not only technical advances, but also stronger political support [42].

- **Mobility sector** with the switch from combustion engine cars to electric vehicles (EVs). They can also contribute to flexibility management through solutions such as vehicle-to-grid (V2G), where vehicles act as temporary batteries for the grid. The last few years have seen improvements in EVs autonomy, greater model availability and enhanced performance. The share of electric cars in total sales is around 18% for 2023 and is expected to continue strongly through 2024 [43]. Sales in some countries have been slow due to typically higher purchase costs compared to conventional vehicles and a lack of charging infrastructure. The EU recently adopted emissions standard for heavy-duty vehicles, which will support electric truck and bus adoption in the coming years.
- **Industry sector** as some industrial processes can be electrified, such as those using low or medium-temperature heat. Heavy industries can resort to solutions such as the use of green hydrogen produced by electrolysis.

The electrification of the loads has major impacts on electricity demand. In particular, an increase in overall demand, as residential, industrial and transport levels become more electrified. Another consequence is the change in end-users consumption profiles. Indeed, the introduction of equipment such as HPs and EVs, is modifying peaks in demand for electricity. For example, electric vehicle charging may be concentrated in the evening or overnight, and electric heating could lead to consumption peaks in winter. This poses a challenge for power grid management, which will have to adapt to these new profiles. Note that equipment such as EVs can help provide flexibility. For instance, EVs could be recharged during periods of high renewable generation, thus avoiding overloading the grid at times of peak demand. This process is spreading across society both earlier and at a faster speed, creating additional capacity needs.

2.4.2. Role of the actors in the energy transition

In a liberalized context, the transition towards a decarbonized electricity system relies on the coordinated commitment of various system actors.

Suppliers and generators

An important issue with the current liberalized model lies in the lack of incentives for generators and suppliers to invest in new generation capacity, especially flexible generation resources (such as gas-fired power plants or storage facilities). This is partly due to the fact that these actors are profit maximizers seeking (overly) high and guaranteed returns on investments, and to a phenomenon often referred to as "missing money": as electricity market prices are too volatile or too low in overcapacity periods (since capped by the regulator), the profitability of the necessary investments is not necessarily ensured. This can lead to a risk of under-investment, resulting in a shortage of capacity in the long-term, especially in periods when renewable energies are not producing sufficient power. Producers may also choose to close unprofitable power plants, increasing the risk of electricity shortages. Many countries are introducing capacity remuneration mechanisms (CRM) to overcome this problematic, with more or less success. These systems do not remunerate energy production, but rather the availability to produce or reduce demand when needed, thus guaranteeing long-term security of supply. CRMs are often financed via electricity tariffs or specific taxes levied on consumers. Hence, these mechanisms offer a solution to ensure that sufficient generation capacity is available when needed, but they require rigorous management to avoid excessive costs for consumers and to avoid disrupting market signals.

End-users

End-users are taking on an increasingly central role in the energy transition. Their engagement and investment in more decentralized, responsible and flexible forms of electricity consumption are crucial. Solutions such as smart meters and energy management systems enable users to better control their consumption. In addition, prosumers can generate their own electricity from renewable sources, while helping to stabilize the grid thanks to domestic storage systems. The regulatory framework around technologies such as batteries, needs therefore to be strengthened. Currently, new modes of exchanges of electricity tend to appear at the local level, which question the market structure. The literature speaks generally of *consumer-centric electric systems*, for which the end-user is placed at the center of the electrical energy supply chain. In recent years, new market mechanisms that have received considerable interest are the *energy communities*, which are more detailed in the next section.

2.5. Energy communities

The European energy and climate targets imply that fossil fuels will have to be replaced by renewable and other low-carbon sources in the next decades [5]. This transition requires a new organization and modernization of the power system, in order to efficiently and reliably manage rising electricity demand, while integrating intermittent renewable energy production and its wide geographic dispersion at all levels of the electricity network. The system is actually facing a massive development of technologies related to distributed energy resources (DERs) (decentralized energy production and storage) and systems for load management and control at the local level. These technological advancements, coupled with a growing environmental awareness of citizens, have led to the emergence of prosumers who are both producers and consumers of electricity. Prosumers can therefore extract or inject electricity into the existing distribution network, allowing them to contribute actively to the grid while also benefiting from self-consumption to reduce dependency on centralized power plants. They can also provide demand-response services by reducing demand during peak times.

The prosumers' involvement is unanimously recognized by the EU as a key to achieve its ambitious environmental targets, and must enable prosumers to fully engage with the system. The reform of electricity markets must therefore involve the integration of prosumers, and enable them to fully engage with the system, so moving towards *prosumer-centric markets*. To this end, the EU introduced the Clean Energy for all Europeans package in 2019 [34], which proposes new rules to enhance the flexibility of the electricity system, reduce carbon emissions and recognize consumer rights on self-generation and to play an active and central role in the electricity markets and the decarbonization of the energy system. The prosumer integration into the energy grid contributes to decentralization and system stability by managing energy flows on a more local scale, which is crucial for absorbing variations linked to intermittent renewable energies. Furthermore, the system benefits from more flexibility and resilience.

Different market mechanisms for increasing the involvement of end-users in the electricity supply chain have been investigated. Some studies propose for instance to keep a centralized market structure, while adapting the wholesale markets to extend their conditions of access to medium and small end-users [44, 45]. Other studies propose a fully decentralized market design involving peer-to-peer trading platforms, such as in [46, 47, 48]. The aim of this approach is to enable local consumers and producers to exchange energy bilaterally, without a centralized supervisory body. An intermediate solution has been formalized by the EU, namely *Energy Communities*, which are discussed below.

2.5.1. Definitions and goals

Through the Clean Energy for all Europeans Package [34], the EU has defined two official entities that formalize energy communities: *Citizen Energy Communities* (CECs), which are described in Directive (EU) 2019/944 [19], and *Renewable Energy Communities* (REC) in Directive (EU) 2018/2001 [6]. They have specific membership criteria, governance requirements and purposes, which focus on the interests of members or the local community rather than financial profit. With communities, the European commission intents to empower citizens and allow them to play an active and central role in the electricity supply chain. More specifically, communities aim at (adapted from [3, 5]):

- Mobilizing private capital for investment in long-lived physical assets needed for the energy transition: actors who do not have the space to invest in renewables can inject their capital into local, collective renewable projects if they engage in a community.
- Unlock LV and MV flexibility provision (e.g., by implementing DSM schemes coordinated at the community level), in order to help the other system actors to ensure the balance and safe operation of the electricity system. Furthermore, it can improve the self-consumption and self-sufficiency of the members. In that way, communities intend to promote a better use of energy resources, in particular those at the distribution level.
- Creating a local stable economic framework, less subject to wholesale price spikes (due e.g., to geopolitical crises).
- Addressing the growing problem of local opposition to the construction of new plants, in particular those based on renewable energy sources.

General definition

Energy communities are new collective actors of the energy system, which gather local consumers, producers and prosumers (i.e., consumers who own their generation assets, and can also produce energy themselves) into organized entities. The directives state that energy communities may engage in activities such as electricity generation, consumption, supply, storage, aggregation, commercial energy services, sharing and selling the energy produced from members' private or community-owned plants. They allow their members to gather their energy resources and exchange locally generated electricity between participants. In this way, members do not depend solely on traditional wholesale/retail markets structure. However, members are free to choose their electricity suppliers for consumption not covered locally and can sell production on conventional markets. Then, energy communities have access to all suitable energy markets both directly and through aggregation in a nondiscriminatory manner, and may have imbalanced responsibility too (although the way the imbalance risk is shared between BRPs and Communities is still subject to debate). Participation in energy communities must be open and voluntary based on transparent and nondiscriminatory criteria, while the effective control can be carried out solely by members that are natural individuals, local authorities and/or small and medium enterprises (SMEs) that are not already active in the energy sector.

All members should enjoy similar governance rights and should have the right to a fair exit procedure if they wish to leave the community [4].

Energy communities can take many forms and be engaged in different activities. Hence, they can face different barriers and provide a wide range of benefits for their members, the local area where it operates, and society in general. Note that they do not have a purely commercial nature, i.e., they do not necessarily provide financial profits for their members. In fact, their primary purpose is to provide environmental (renewable energy promotion, greenhouse gas emission reduction, etc.), economic (energy bill reduction, partial protection to wholesale and retail markets prices spike, etc.) and/or social (enhanced social inclusion, energy poverty reduction, etc.) community benefits to its members or to the local areas where it operates. Furthermore, energy communities with local energy exchanges may bring benefits to the distribution network provided that collective demand-side management schemes, implicitly or explicitly aware of grid technical constraints (lines losses and congestion, voltage management to avoid PV tripping, lower peaks at the substation transformers, etc.), and appropriate regulation (e.g., grid tariffs), are implemented.

Comparing RECs and CECS

The two official entities that formalize energy communities, namely CECs and RECs, are closely related and refer to a way of organizing citizens who want to cooperate together in an electricity supply activity, based on open and democratic participation and governance, while providing (potentially) non-profit benefits to the members or the local community. They have, however, important differences.

The RECs criteria are more stringent than CECs. Renewable Energy Communities membership is forbidden to large enterprises for instance. Furthermore, the REC should be effectively controlled by members located in the proximity (e.g., from an electrical point of view [49] or from a geographical one [50]) of the renewable projects that are owned or are developed by the REC [5]. The RECs must restrict their local energy production and sharing to renewable energies. They can engage with various energy carriers, such as heat or biogas, in parallel with electricity.

Unlike RECs, the CECs membership is not restricted to specific types, but any large member company, including those interacting directly with markets, cannot exercise control over the community. On the other hand, controlling members of CECs can be dispersed over vast areas with no geographical restrictions. Additionally, CECs are limited to electricity activities, but they

Figure 2.9.: Example of a Renewable Energy Community.

can involve non-renewable energy sources. Furthermore, they can provide energy efficiency services and charging services.
In this thesis, we focus on Renewable Energy Communities as presented in Figure 2.9.

Implementation in member states

The EU legislation only sets the direction that national laws and regulations must follow. Member states benefit from freedom to implement the framework practically; therefore the development path of energy communities differs across countries, extending the heterogeneous situation that is currently visible in Europe [5]. Member states have to clarify in their specific legal and regulatory framework what legal form a CEC and REC can take. More particularly, they shall define: 1) the purpose of the community; 2) market access to avoid discrimination of energy communities and other new players in the energy system; 3) electricity sharing rules within the community that are transparent; 4) financially sustainable network tariffs; 5) what proximity of the community's renewable project means and 6) a democratic governance system [4].

Some countries have already transposed the directive, or are currently in the process of transposition into regional or national decrees, legal frameworks, etc [51]. Germany defines energy cooperatives and citizen energy cooperatives in the 2017 Renewable Energy Act [52]. The French law on Collective Self-Consumption dates back to 2017 [53], while Italy tackled the issue with its law 08/2020 in February 2020 and later evolutions (Law 199/2021). The Greek law on energy communities was published in 2018 [54]. The Netherlands provides the legal definition of RECs, and Spain defined local energy communities in 2019 Royal Decree 244/2019. In Wallonia in Belgium, a decree first published in 2019, reviewed in 2022, and a government edict was released in March 2023 [55]. The science and technology sectors have, in parallel, launched many initiatives to implement pilot projects of energy communities, such as for instance the E-Cloud project led by ORES (one of the main Walloon DSO) [56]. We refer to [57] for many other examples of projects in Europe.

More specifically, the EU let the freedom to the member states to impose or not specific energy exchange mechanisms (or set of rules) within the energy community. This led to many propositions from the public, private and academic sides, which are discussed below.

2.5.2. Market designs

Among all the potential energy exchange mechanisms, two main frameworks stand out: the organization of a local energy market (see e.g., [58, 59]), and the rule-based dispatch of energy community surplus (see e.g., [29, 60, 61]), in application in Wallonia and France for instance.

The principle of a local energy market within energy communities is similar to the traditional market structures, where participants make demand and offer bids based on their expected net load [58, 59]. The literature provides a classification of these local energy markets. A community-based market is an internal market where the trading activities are managed by a central operator, called a community manager (CM) [62, 63, 64, 65] which clears the markets based on member bids. In addition, the community manager is an intermediate between the community and the rest of the system and could provide potential new services for the upstream networks. These markets enhance members' involvement and cooperation to share common good, but are accompanied by difficulties such as managing the expectations and preferences of participants at all times and ensuring fair and impartial sharing of energy between them. A distributed peer-to-peer (P2P) market design allows peer-to-peer energy exchanges inside between members of the community [66, 67, 68, 69, 70]. Two members can agree on a transaction for a certain amount of energy and a price without centralized supervision [48]; furthermore, energy is treated as a heterogeneous product with some characteristics, in which participants can express their preferences [67]. This approach increases the empowerment of active consumers and provides energy use aligned with each agent's preferences. However, this brings requirement issues in terms of communication infrastructure and maintenance to achieve scalability. We can expect a potentially slow convergence to reach consensus on the final supply of energy and the lack of centralized control can result in difficult predictions of system behavior.

However, the main assumptions of perfect competition are hardly met in communities with local markets. For instance, the limited size of RECs with

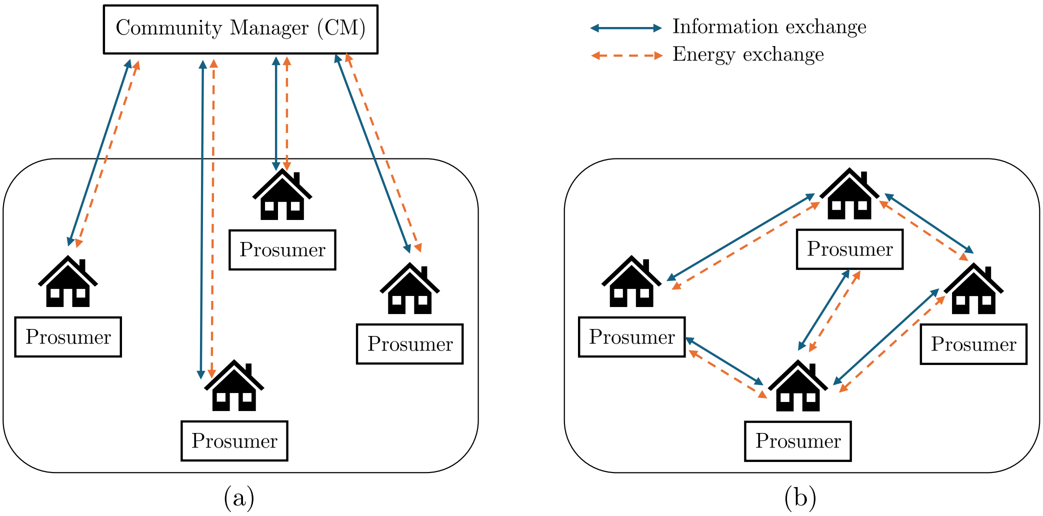


Figure 2.10.: Internal market designs: (a) community-based, and (b) peer-to-peer.

internal markets tends to inherently provide market power to some participants. In [71], the authors propose an alternative design of energy communities that leverages prosumer empowerment and mutualization of excess resources to optimize a better reflection of the true electricity total cost. It implements a collaborative demand-side management scheme inside a community that aims to optimize the use of resources and energy exchanges by unlocking some flexibility, in order to achieve the best objective. This approach avoids issues related to market-based transactions and provides communities in which cooperation prevails over competition [72]. The present thesis focuses on this specific category of market design, and we provide a full description of the REC framework in Chapter 4.

2.5.3. Research challenges and scope of the thesis

Different time horizons must be considered for favoring the uptake of energy communities, ranging from long-term (with investment and sizing problems) to short-term (operational management: collective demand-side management, deviations settlement, etc.) as presented in Figure 2.11. In fact, implementing renewable energy communities poses a variety of challenges, described below.

The first challenge we address in this thesis lies in the explicit modeling of the strategic interactions between members, which may lead to different equilibrium situations. Also, we focus on the preferences of community members, which

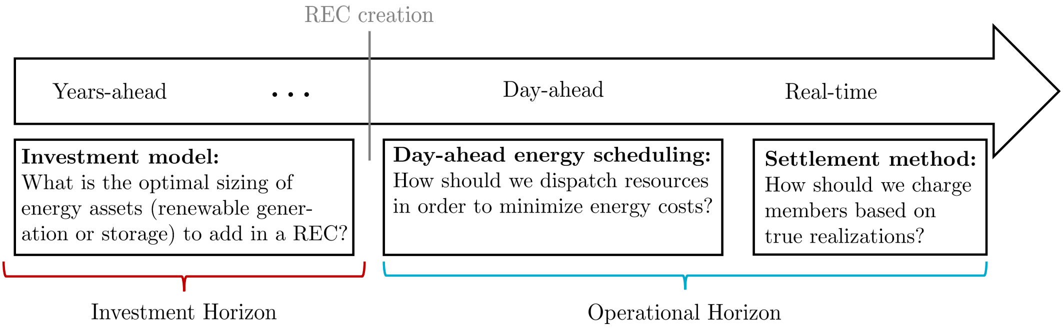


Figure 2.11.: Diagram of the different time horizons in the operations of a renewable energy community.

may be different within a single community: for instance, a member A may use her energy assets in order to minimize her own energy bill only, whereas a member B will act to reduce the GHG emissions of the whole community, even if her energy bill is slightly increased. A third member C may also be ready to pay more for ensuring the availability of the electricity supply one hundred percent of the time, which can be important in regions and countries subject to reliability issues and/or for activities involving uninterruptible processes). To that end, in this thesis, we resort to game theory while studying more specifically the (day-ahead) energy scheduling horizon, as well as the investment planning horizon.

Energy exchange scheduling aims to coordinate the energy assets of community members, providing optimal recommendations on energy consumption and exchanges. This process uses the temporal flexibility (such as flexible appliances and ESS) of the members to adjust demand to production in order to achieve the best objective of the community. The most relevant problems are long-term investment planning and day-ahead scheduling. We call *day-ahead energy resources scheduling*, the energy exchange scheduling for one-day ahead horizon. This horizon corresponds to day-ahead electricity markets, where the essential adjustments for balancing demand and production are made (see subsection 2.3.1). Besides, end-users generally have a good idea of their energy requirements for the following day, which makes it easier to forecast consumption. Scheduling one day in advance is then highly relevant and practical.

In general the implementation of energy planning depends on how the costs are distributed among the members' individual bills. It is crucial to define a fair and efficient allocation method to ensure the highest possible commitment from all members. This has already been investigated in the literature, whether in terms of fairness, user incentives according to their profile, self-consumption and flexibility reward or the efficiency [29, 73, 74, 61]. The cost-based billing design distinguishes three main frameworks: an endogenous approach via non-cooperative games (Nash [75, 76] or Stackelberg games [12, 13]), coalition games [16, 61, 69] and ex-post allocation [29, 74, 61].

In the first part of the present work, we focus on electricity-only Renewable Energy Communities established on the public electricity distribution network. The members of the community are equipped with bi-directional metering devices, or smart meters, which monitor energy flows and ICT systems for implementing intelligent algorithms managing resources. Furthermore, each member can own flexible appliances, local renewable generation and electricity storage assets (typically battery energy storage systems). Chapter 4 mathematically models the interdependence between community members who share common resources, and allocates costs between members via four distribution methods already mentioned in [76].
Secondly, we address the problem of investments in assets within a community by tackling a subject that is still largely unexplored in European directives and in scientific literature: the RECs management if users decide to join or leave the community during the life of the community with potential investments. We augment the energy exchange scheduling problem developed in Chapter 4 to address this issue in Chapter 6.

The second challenge we address in the present thesis is the bounded rationality of community members. Most of the literature adopts, indeed, the hypothesis of the rationality of economic agents, and relies more generally on the Expected Utility Theory (EUT) [8]. The rationality of an individual designates her ability to make coherent decisions to optimize her objectives, preferences or utility, based on available relevant information and unlimited processing capacity. However, there are many experimental evidence that people are not rational in uncertain and risky situations [77, 78, 79]. In particular, people are bad to estimate probabilities. Thus rational models, such as EUT, are not adequate for modeling real decision-making processes under uncertainty, such as DSM or investment planning problem [80]. As a consequence, Prospect Theory (PT), a theory of behavioral economics that questions the traditional EUT and was introduced by Kahneman and Tversky in 1979 [77, 81], has been proposed as a promising framework for modeling the non-rational energy preferences of end-users. It has recently been employed in the energy sector in [13], where the authors used PT to model the risk behavior of prosumers who face future uncertain energy prices. This research challenge is addressed in the second part of this thesis, dealing with the long-term investment horizon.

Other research challenges in the context of energy communities are also encountered in the literature. The most significant in our opinion are exposed below:

- Investment models and strategies (collective investment, individual investment with sharing of individual surplus, etc.) in communities [82, 83, 28].
- Multi-energy communities, e.g., also involving the heat vector [65].
- Interaction with market actors outside the community (e.g., Balance Responsible Parties, Flexibility Service Providers, etc.) [84, 13].
- Techno-economic impacts on the distribution grid: energy communities with local energy exchanges can bring benefits to the distribution networks [28], provided that grid tariffs are carefully designed. A study of how inadequate energy community network tariffs can lead to an excess of energy community adoption and non-desirable outcomes for the grid, can be found in [15].
- Multi-objective communities: some communities may indeed combine multiple objectives [85], such as reducing the global electricity bill and the carbon footprint of the REC as much as possible, and/or reduce their dependency on the external system by efficiently using their production and consumption means, so as to reach optimal collective self-consumption and/or self-sufficiency.
- Dealing with uncertainty (of local generation, consumption, of asset parameters, of members' behaviors, etc.) in communities [86, 87, 88]
- Social acceptance [89, 90, 91, 92].

Part I.

Strategic Games for Day-Ahead Scheduling

CHAPTER 3.

Mathematical Fundamentals

In Chapter 2, we have introduced the concept of renewable energy communities, whose implementation poses a variety of challenges. This thesis focuses on communities built around a collaborative demand-side management scheme. In the first part of this work, we address the day-ahead energy exchange scheduling problem. This short-term horizon allows for the determination of an operational plan for the next day while also providing a foundation for evaluating long-term operational costs in investment planning, which are investigated in the second part of the work (see Chapter 6).

The day-ahead energy resources scheduling problem can be treated from different perspectives. An intuitive approach is to consider the problem as a centralized model, where a single central operator (e.g., a community manager) is empowered to optimize an objective function representing the REC's global interests (e.g., minimizing the total cost). In practice, however, communities bring together consumers and prosumers who are more likely to pursue selfish optimization of their individual goals. Furthermore, strategic interaction can emerge between members sharing common resources (e.g., competing for network usage, energy pool, limited energy storage capacity, etc.). These interdependence in objective functions and possibly available member actions, directly influence operational decisions and scheduling outcomes of the energy exchange scheduling problem. These observations lead to decentralized formulations of the problem. Then, complex interactions between rational members are modeled through *noncooperative game theory*. These models can be analyzed and solved using various mathematical tools, such as *convex optimization*, *variational inequality theory* and *potential games*. In particular, potential games present a framework for identifying equilibria that align individual incentives with community objectives, thus establishing a connection between decentralized decision-making and community efficiency.

This chapter aims to cover only definitions and concepts essential for the comprehension of the models' development in this thesis. Section 3.1 introduces the principles of convex optimization problems, which are a basis for analyzing and solving centralized formulations. Section 3.2 defines normal-form games and Nash equilibrium, alongside measures for evaluating Nash equilibria efficiency. These two sections are fundamental to understanding the various model formulations presented in Chapter 4. Whereas, the subsequent sections provide advanced tools for in-depth analysis and resolution of the problems. Then, variational inequality theory and potential games are presented in Section 3.3. Finally, Section 3.4 presents the algorithms used in this work and discusses their convergence properties.

3.1. Convex optimization problems

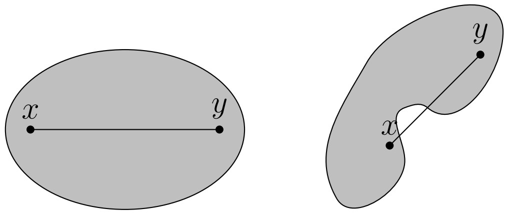
Mathematical optimization is a discipline that studies problems involving the optimization of a given function in a specified space. Convex optimization problems are a subclass of such problems, characterized by the minimization of a convex objective function (or the maximization of a concave function) over a convex set. This framework guarantees that any local solution is also global, making these problems more accessible to analyze and to be solved using well-developed algorithms. It can be used to model problems in a wide range of disciplines: economics, engineering, machine learning, etc. Convex optimization, therefore, relies on convex analysis [93]. In this section, we introduce the concepts of convex optimization problems, which serve as a basis for the various models presented throughout this thesis. This section is based on [94, 95].

3.1.1. Basic definitions

We respectively denote by $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, and $\mathbb{R}$ the sets of natural numbers, integers, rational numbers, and real numbers. We define by $\mathbb{R}^n$, the Euclidean n-dimensional space, $n \in \mathbb{N}$. We identify $\mathbb{R}^n$ with $\mathbb{R}^{n\times 1}$. For every $x \in \mathbb{R}^n$, we denote by $x^\top$ the corresponding element in $\mathbb{R}^{1\times n}$. The usual inner product of two vectors $x, y \in \mathbb{R}^n$ is then denoted by $\langle x, y\rangle = x^\top y = \sum_{i=1}^n x_i y_i$. The norm associated with the scalar product is defined by $\|x\| = \sqrt{x^\top x}$ for all $x \in \mathbb{R}^n$.

Convexity

We introduce definitions of convexity for sets and functions, which are typically required for the optimization solvers used in this work. We start with the



(a) Convex (b) Non-convex

Figure 3.1.: Graphical interpretation of convex sets.

definition of a convex set.

Definition 3.1. A set $\Omega \subseteq \mathbb{R}^n$ is *convex* if the line segment between any two points x, y in Ω, belongs to Ω, i.e.

$$\forall x, y \in \Omega \; \forall \alpha \in [0,1], \quad \alpha x + (1-\alpha)y \in \Omega. \tag{3.1}$$

The Figure 3.1 shows illustrative examples of the definition. The intersection of any collection of convex sets is a convex set.

We now define three concepts of convex function.

Definition 3.2. Let $\Omega \subseteq \mathbb{R}^n$ a convex set, a function $f : \mathbb{R}^n \to \mathbb{R}$, with $\mathrm{dom} f = \Omega$, is said to be

- *convex* on Ω if, for all $x, y \in \Omega$ and $\alpha \in [0,1]$,

$$f(\alpha x + (1-\alpha)y) \leqslant \alpha f(x) + (1-\alpha)f(y) \tag{3.2}$$

- *strictly convex* on Ω if, for all $x, y \in \Omega$ such as $x \neq y$ and $\alpha \in]0,1[$,

$$f(\alpha x + (1-\alpha)y) < \alpha f(x) + (1-\alpha)f(y) \tag{3.3}$$

- *strongly convex* on Ω with parameter $m > 0$ if, for all $x, y \in \Omega$ and $\alpha \in [0,1]$,

$$f(\alpha x + (1-\alpha)y) \leqslant \alpha f(x) + (1-\alpha)f(y) - \frac{m}{2}\alpha(1-\alpha)\|x-y\|^2. \tag{3.4}$$

The inequality (3.2) requires that the line segment between any two points $(x, f(x))$ and $(y, f(y))$, lies above the graph of f. A function f is *concave* if $-f$ is convex and *strictly concave* if $-f$ is strictly convex. Obviously, an affine function always holds equality in 3.2, so all affine and linear functions are both convex and concave. Strict convexity means that the graph of f lies below

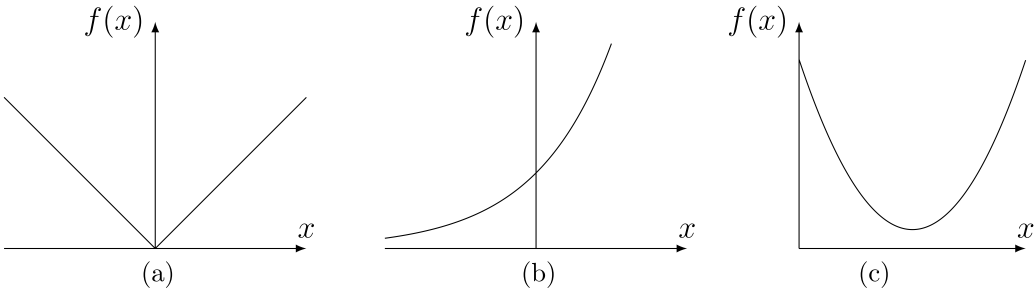


Figure 3.2.: Some examples of convex functions: (a) $f(x) = |x|$ is a convex function (without being strictly convex), (b) $f(x) = \exp(x)$ is a strictly convex function (without being strongly convex) and (c) $f(x) = (x - 3/2)^2 + 1/4$ is a strongly convex function.

the segment, while strong convexity implies the graph of f to lie "sufficiently" below the line segment. The Figure 3.2 shows illustrative examples of these definitions.
It is easy to see from the definitions 3.2 that the following relations hold:

Let $\Omega \subseteq \mathbb{R}^n$ a convex set, a function $f : \mathbb{R}^n \to \mathbb{R}$, with $\text{dom} f = \Omega$.

$$\text{strongly convex} \Rightarrow \text{strictly convex} \Rightarrow \text{convex}$$

The converse of neither implication is true. It can be observed on Figure 3.2 that $f(x) = |x|$ is convex but not strictly convex, and $f(x) = e^x$ is strictly convex but it is not strongly convex, since the second derivative can be arbitrarily close to zero.

Hyperplanes and half-spaces

Definition 3.3. A *hyperplane* in $\mathbb{R}^n$ is a set of the form

$$H = \{x \mid a^\top x = b\}, \tag{3.5}$$

with $a \in \mathbb{R}^n$, $a \neq 0$ and $b \in \mathbb{R}$.

Analytically, it is the solution set of a linear equation among the components of x. Geometrically, the hyperplane H can be interpreted as the translation, along direction a of the set of points that are orthogonal to a, the constant b determines the offset from the origin.

If $x_0 \in H$, then for any other element $x \in H$, we have $a^\top x_0 = a^\top x = b$. Hence the hyperplane can be characterized as the set of vectors x such that $x - x_0$ is orthogonal to vector a :

$$H = \{x \mid a^\top (x - x_0) = 0\}.$$

A hyperplane divides $\mathbb{R}^n$ into two half-spaces.

Definition 3.4. A *half-space* is the solution set of one linear inequality

$$D = \{x \mid a^\top x \leqslant b\}, \tag{3.6}$$

$a \in \mathbb{R}^n,\ a \neq 0$, and $b \in \mathbb{R}$.

The half-space can also be expressed as

$$D = \{x \mid a^\top (x - x_0) \leqslant 0\},$$

where x_0 is any point on the associated hyperplane, then $a^\top x_0 = b$. Geometrically, a half-space is the set of points that form an obtuse angle with the vector a. The boundary of a half-space is the associated hyperplane.

Polyhedra

Definition 3.5. A *polyhedron* is the intersection of a finite number of half-spaces and hyperplanes, so it is defined as the solution set of a finite number of linear (or affine) equalities and inequalities :

$$\begin{aligned} \mathcal{P} &= \{x \mid a_i^\top x \leqslant b_i,\ c_j^\top x = d_j,\ i = 1, \ldots, m,\ j = 1, \ldots, p\}, \\ &= \bigcap_{i=1}^{k} D_i \cap \bigcap_{j=1}^{l} H_j, \end{aligned} \tag{3.7}$$

where $D_1, \ldots, D_k \subseteq \mathbb{R}^n$ are half-spaces, and $H_1, \ldots, H_l \subseteq \mathbb{R}^n$ are hyperplanes.

A polyhedron is the intersection of half-spaces and hyperplanes, which are convex, and convexity is preserved under the intersection, so polyhedra are convex sets. A *polytope* is a bounded polyhedron, but some authors use the opposite convention.

Polyhedra are essential elements in the formalization of convex optimization problems. They allow us to describe the admissible sets on which these problems are defined, and to characterize the properties of optimal solutions.

3.1.2. Optimization problems

An optimization problem is a mathematical entity that enables the minimization (or maximization) of a specific objective or preference, subject to diverse physical, financial, or other limitations. A variety of practical problems involving decision making, system design, analysis and operation can be cast in the form of a mathematical optimization problem. We present the definition of a constrained optimization problem.

Definition 3.6. Let $f : \mathbb{R}^n \to \mathbb{R}$ with $g_i : \mathbb{R}^n \to \mathbb{R}$, for $i = 1, \dots, m$ and $h_j : \mathbb{R}^n \to \mathbb{R}$, for $j = 1, \dots, p$.
A *constrained optimization problem*, noted $(f, (g_i)_{i=1}^m, (h_j)_{j=1}^p)$, has the standard form written as

$$\begin{aligned} \min_{x \in \mathbb{R}^n} \ & f(x) \\ \text{s.t.} \ & g_i(x) \leqslant 0, \quad i = 1, \dots, m \\ & h_j(x) = 0, \quad j = 1, \dots, p. \end{aligned} \tag{3.8}$$

The vector $x = (x_1, \dots, x_n)$ is the set of *decision variables* of the problem, the function f is the *objective function* to be minimized. The inequalities $g_i(x) \leqslant 0$ are called *inequality constraints* and the equations $h_j(x) = 0$ are the *equality constraints*. The functions g_i and h_j are respectively the *inequality and equality constraints* functions.

Example 3.1 (Electricity-production problem [96])**.** An electricity producer has two generation plants that have capacities of 12 and 16 units per hour, respectively. These plants share a common thermal management system, which imposes some operational constraints. Specifically, the sum of the hourly output of plant 2, and twice the one from plant 1, must be at least 8 units. In addition, the sum of the hourly production of plant 2 and two thirds of the one from plant 1 must be no more than 18 units. The generator charges 1€ per unit per hour for the electricity produced. He seeks to determine the optimum production level total for both plants, in order to maximize hourly revenues.

The optimization problem formulation can be written as

$$\begin{aligned}
\min_{x_1,x_2} \quad & -x_1 - x_2 \\
\text{s.t.} \quad & -2x_1 - x_2 + 8 \leqslant 0 \\
& \frac{2}{3}x_1 + x_2 - 18 \leqslant 0 \\
& x_1 - 12 \leqslant 0 \\
& x_2 - 16 \leqslant 0 \\
& -x_1 \leqslant 0 \\
& -x_2 \leqslant 0.
\end{aligned}$$

Note that minimizing a function f is equivalent to maximizing $-f$.

We define Ω a *feasible set* as the set of decision variables that are feasible, i.e., that satisfy the optimization problem's constraints

$$\Omega := \{x \mid g_1(x) \leqslant 0, \dots g_m(x) \leqslant 0, h_1(x) = 0, \dots, h_p(x) = 0\}. \tag{3.9}$$

If the feasible set is empty, the problem is said to be infeasible. When we solve an optimization problem, we want to find a global minimum, but for most problems, we can only find some local minima. Let $B(x, r) = \{y \in \mathbb{R}^n : \|x - y\| \leqslant r\}$ be a ball centered at point x with radius r. A feasible point x^* is a local minimum if there exists $\rho > 0$ such that $f(x^*) \leqslant f(x)$ for all $x \in \Omega \cap B(x^*, \rho)$. A feasible point $x^* \in \Omega$ is a global minimum if $f(x^*) \leqslant f(x)$ for all $x \in \Omega$. Then, a local optimum minimizes the objective function's value among neighboring feasible points, but is not guaranteed to have a lower objective value than all other points in the feasible set as a global minimum. Clearly, a global minimum is also a local minimum, but the opposite may not be true. We define the *optimal set* X_{opt} as the set of all global optimal points of the optimization problem.

This thesis considers a subfield of optimization problems called convex optimization problems.

Definition 3.7. An optimization problem $(f, (g_i)_{i=1}^m, (h_j)_{j=1}^p)$ (3.8) is a *convex optimization problem* if the objective function f and inequality constraints functions g_i are convex and equality constraint functions h_j are affine.

The feasible set Ω of a convex optimization problem is convex, since it is an intersection of convex sets. Then, in a convex optimization problem, we minimize a convex objective function over a convex set. Convex optimization problems have the fundamental property that any local optimum is also a global optimum. Furthermore, their advantageous mathematical structure and the availability of efficient solution methods make these problems tractable [94].

Two classes of convex problems are considered in this work.

Definition 3.8. The optimization problem $(f, (g_i)_{i=1}^m, (h_j)_{j=1}^p)$ (3.8) is called a *Linear Programming Problem* (LP) if the functions involved are all affine.

Hence, a LP minimizes an affine function over a polyhedron. The two most well-known solution algorithms are the simplex algorithm and the interior-point method [96, 94].

Definition 3.9. The optimization problem $(f, (g_i)_{i=1}^m, (h_j)_{j=1}^p)$ (3.8) is called a *Quadratic Programming Problem* (QP) if the objective function is convex quadratic and the constraint functions are all affine.

Then, a QP minimizes a convex quadratic function over a polyhedron. Most solvers use an extension of the simplex algorithm or an extension of the interior-point method [97, 94].

3.1.3. Optimality conditions

We remind the reader that we do not assume that the problem (3.8) is convex, unless explicitly stated. The *optimality conditions* provide rigorous criteria that a solution to an optimization problem needs to check to be optimal (usually necessary but not sufficient). These conditions are powerful theoretical tools for guiding the design (e.g., stopping criterion) and analysis (e.g., convergence) of solutions algorithms, and for grasping the solutions structure. They can also be used in the evaluation process of the optimal solutions (sensitivity analysis).

This section is limited to optimality conditions for finite-dimensional differentiable optimization problems (in Fréchet's sense). We recall the definition for the gradient of a differentiable function at a point.

Definition 3.10. Let $a \in \mathbb{R}^n$ and $f : \mathbb{R}^n \to \mathbb{R}$ differentiable in a, the gradient of the function f at the point a is

$$\nabla f(a) := \left(\frac{\partial f}{\partial x_1}(a), \ldots, \frac{\partial f}{\partial x_n}(a) \right).$$

The minimum principle

The central optimality condition is called the *minimum principle*.

Theorem 3.1. *Consider an optimization problem* (3.8) *with* $f : \mathbb{R}^n \to \mathbb{R}$ *differentiable, and the feasible set* Ω *is convex.*

- *if* x^* *is a local minimum of* f *over* Ω*, then*

$$(x - x^*)^\top \nabla f(x^*) \geqslant 0, \ \forall x \in \Omega. \tag{3.10}$$

- *if* f *is convex, then the condition is sufficient for* x^* *to be a global minimum.*

This condition states that the directional derivative of the objective function, the left term of (3.10), should be nonnegative for all feasible directions. The minimum principle is illustrated in Figure 3.3.

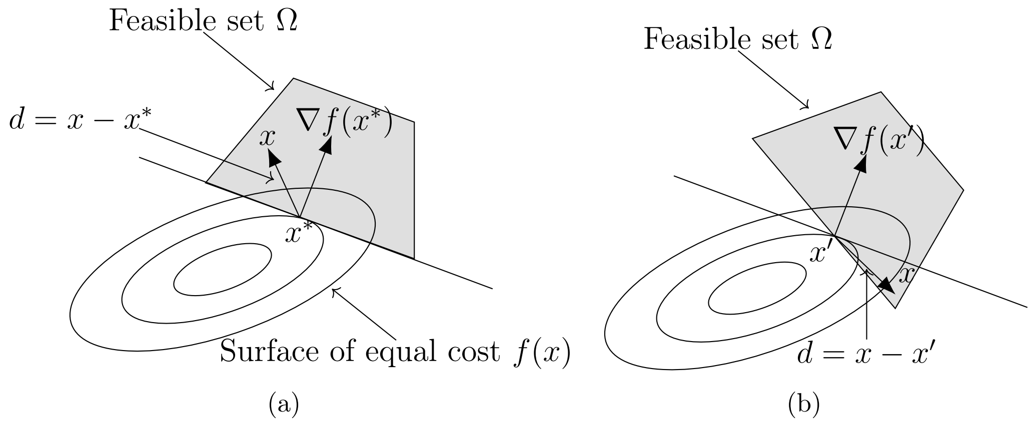


Figure 3.3.: Geometric interpretation of the minimum principle: (a) a feasible point x^* that satisfies the minimum principle, $\nabla f(x^*)$ form a non-obtuse angle with all feasible vector d originating from x^*; and (b) a feasible point x' that does not, there exists other feasible point $x \neq x'$ such that $f(x) < f(x')$.

If the feasible set Ω is defined by inequalities and equalities, other optimality conditions deserve to be discussed, the *Karush-Kuhn-Tucker (KKT) conditions* [98].

Karush-Kuhn-Tucker conditions

The Karush-Kuhn-Tucker conditions is a system of equations and inequalities forming what is known as a Mixed Complementary Problem (MCP) [99, 100]. Generally used to analyze solutions, they can even, in some cases, be solved to obtain a closed-form analytical solution. Furthermore, they are the foundation of numerical techniques for resolving optimization problems. To formulate the KKT conditions, it is convenient to define a *Lagrangian function.*

The Lagrangian function of an optimization problem $(f, (g_i)_{i=1}^m, (h_j)_{j=1}^p)$ in (3.8) is

$$L(x, \lambda, \mu) = f(x) + \sum_{i=1}^{m} \lambda_i.g_i(x) + \sum_{j=1}^{p} \mu_j.h_j(x), \tag{3.11}$$

where functions f, g_i and h_j are continuously differentiable in the feasible region. The scalars λ_i and μ_i are called Lagrange multipliers. The KKT conditions of problem (3.8) allows us to find decision variables (primal variables) $x \in \mathbb{R}^n$ and Lagrange multipliers (dual variables) $\lambda \in \mathbb{R}^m$, and $\mu \in \mathbb{R}^p$, such that

$$\nabla_x f(x) + \sum_{i=1}^{m} \lambda_i \nabla g_i(x) + \sum_{j=1}^{p} \mu_j \nabla h_j(x) = 0 \tag{3.12a}$$

$$h_j(x) = 0, \ \forall j \in \{1, \dots, p\} \tag{3.12b}$$

$$g_i(x) \leqslant 0, \ \forall i \in \{1, \dots, m\} \tag{3.12c}$$

$$\lambda_i \geqslant 0, \ \forall i \in \{1, \dots, m\} \tag{3.12d}$$

$$\lambda_i.g_i(x) = 0, \ \forall i \in \{1, \dots, m\}. \tag{3.12e}$$

where ∇_x defines the gradient with respect to x. Condition (3.12a) requires that the gradient of the Lagrangian function (3.11) is equal to zero for an optimal solution x. Conditions (3.12c)-(3.12e) can be compactly written as $0 \leqslant \lambda \perp g(x) \leqslant 0$, where the "perp" operator $\perp$ means the inner product of two vectors equal to zero. We often use this notation in the thesis.

Theorem 3.2. *Let an optimization problem* (3.8) *where functions* f, g_i *and* h_j *are continuously differentiable.*

- *If* x^* *is a local minimum of this problem and satisfies some regularity conditions* [a]*, then there exist Lagrange multipliers* $\lambda^* \in \mathbb{R}^m$ *and* $\mu^* \in \mathbb{R}^p$ *such that* (x^*, λ^*, μ^*) *satisfies KKT conditions* (3.12)*.*
- *If the optimization problem is convex and there exists* (x^*, λ^*, μ^*) *which satisfies* (3.12)*, then* x^* *is a global optimal solution.*

[a] e.g., LICQ or SQ

The first implication states that KKT conditions are necessary, providing the conditions that a regular local optimum must fulfill. They are also *sufficient* to be a global optimum if the optimization problem is convex, translated by the second implication. This theorem shows that, under some additional conditions, the minimum principle is equivalent to the KKT conditions for convex optimization problems.

To guarantee the validity of the first point of the theorem, an additional assumption must be made at an optimum x^* for it to be a stationary point in the Lagrangian. This assumption is that a constraint qualification must be held at x^*. There exist many mathematical conditions that ensure this. We introduce two of them. One of the most widely used and well-known constraint qualifications is the linear independence constraint.

Linear Independence Constraint Qualification (LICQ) is held for x if $\nabla g_i(x)$ for all $i \in \mathcal{I}(x)$ and $\nabla h_j(x)$ for all $j \in \{1, \ldots, p\}$ are linearly independent, with $\mathcal{I}(x) := \{i \mid g_i(x) = 0\}$ the set of constrained active inequality indices.

Note that if a local minimum satisfies LICQ, then Lagrange multipliers are unique.

If (3.8) is a convex optimization problem, then Slater's condition is generally used.

Slater's Constraint Qualification (SQ) holds for a convex optimization problem if there exists a point $\overline{x}$ in the relative interior of the convex set Ω for which $g_i(\overline{x}) < 0$ for all $i \in \{1, \ldots, m\}$ and $h_j(\overline{x}) = 0$ for all $j \in \{1, \ldots, p\}$.

Remark 3.1. The existence of multipliers under affine constraints in a local minimum does not require any additional assumptions. Then, the KKT conditions are necessary and sufficient for LP and QP problems.

3.2. Normal-form games

The centralized problems, such as global optimizations, are based on a single objective function and do not take into account the fact that stakeholders may potentially be strategic. In this case, stakeholders act as selfish rational agents, choosing actions to optimize their individual objectives, which could conflict with the other agents' objectives. So, to model these essential strategic interactions, we can formalize energy exchange scheduling inside a REC, in the game theory framework.

Game theory is a mathematical field that studies strategic interactions between rational stakeholders. Based on the expected utility theory of Von Neumann and Morgenstern [8], this theory has many applications in economics, social sciences, biology, politics, computer sciences and energy systems. In this thesis, we focus on noncooperative games, which model situations where players move independently to optimize their individual outcomes. This section introduces the theoretical concepts used in this work, based on [7].

3.2.1. Strategic games and Nash equilibrium

In the first part of this thesis, we study a strategic interaction model called *strategic game*. This model includes a set of players and specifies for each of them a set of possible strategies (actions) and the costs (or payoffs) associated with each possible outcome. We provide a formal definition of a normal-form game [7].

Definition 3.11. A *strategic (or normal-form) game* is defined by the tuple $G = (\mathcal{N}, (\mathcal{S}_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$ where:

- $\mathcal{N} = \{1, \ldots, N\}$ is a finite set of N players.
- $\mathcal{S} = \prod_{i\in\mathcal{N}} \mathcal{S}_i$ is the set of all game strategy profiles with $\mathcal{S}_i$ the player i's strategy set, for all $i \in \mathcal{N}$.
- The function $u : \mathcal{S} \to \mathbb{R}^N$ is composed of N-functions $u_i : \mathcal{S} \to \mathbb{R}$, where u_i is the player i's cost (or payoff) function for all $i \in \mathcal{N}$.

A vector of strategies $s = (s_1, \ldots, s_N) \in \mathcal{S} = \prod_{i\in\mathcal{N}} \mathcal{S}_i$ is referred to as a strategy profile, and as an outcome of the game. Given a player $i \in \mathcal{N}$, we note the set of other players' strategies as $\mathcal{S}_{-i} := \prod_{j\neq i}^{N} \mathcal{S}_j$. Then $s_{-i} \in \mathcal{S}_{-i}$, such as $s_{-i} = (s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_N)$. To emphasize the i-th player's variables within $s \in \mathcal{S}$, we write (s_i, s_{-i}). If the strategic set $\mathcal{S}_i$ of every player is finite, then the normal-form game is finite.

A normal-form game represents an interaction between selfish players, in which each player $i \in \mathcal{N}$ chooses a strategy from her set $s_i \in \mathcal{S}_i$, simultaneously without communicating. This gives a strategy profile $s = (s_i)_{i\in\mathcal{N}}$ and each player i receives the cost $u_i(s_i, s_{-i})$. All players know the rules of the games, but there is no information about the choices made by the other players. The player i's only goal is to minimize her cost function, and he is not interested in minimizing or maximizing her opponents' costs. However, the cost of each player depends not only on her own actions, but also on the strategies taken by the other players, which explains that the function of each decision-maker $i \in \mathcal{N}$ is defined over $\mathcal{S}$ rather than $\mathcal{S}_i$. In summary, a strategic game models a situation where all players choose their strategies independently and simultaneously, once and for all.

We can represent a finite strategic game as an N-dimensional matrix of payoffs, as illustrated by the famous prisoner's dilemma in Example 3.2 with Table 3.1. However, if the set of actions is not finite, then the strategic game is continuous, which is not presentable via matrices, see Example 3.4.

Example 3.2 (Prisoner's dilemma)**.** Two suspects in a crime are put into separate cells and want to minimize their jail term. If they both confess (C), they will be sentenced to two years in prison. If only one of them confesses, he will be free, while the other will receive a sentence of three years. If both remain silent (S), they will both spend one year in prison. There are two players $\mathcal{N} = \{1, 2\}$ and each player has only two actions available: $\mathcal{S}_1 = \mathcal{S}_2 = \{\text{C, S}\}$. We have the following game representation.

		Suspect 2	
		C	S
Suspect 1	C	$(2, 2)$	$(0, 3)$
	S	$(3, 0)$	$(1, 1)$

Table 3.1.: The prisoner's dilemma.

Example 3.3 (Battle of Sexes (BoS))**.** A couple wants to go out tonight. Caroline would like to see a movie, while Eric is more interested in attending a football match. Both would rather go out together than alone, but each prefers her own activity to that of her partner. The players' preferences are expressed via payoffs as follows. In this context, the players want to maximize their payoffs.

		Eric	
		Movie	Football
Caroline	Movie	$(2, 1)$	$(0, 0)$
	Football	$(0, 0)$	$(1, 2)$

Table 3.2.: Battle of sexes game.

Example 3.4 (Cournot duopoly)**.** Two energy producers $\mathcal{N} = \{1, 2\}$ are economically rational and compete with each other and choose q_i the amount of electricity generated to maximize their profits. The selling price is a decreasing function of the total market production $p(q_1, q_2) = \alpha - \beta.(q_1 + q_2)$, with $\alpha, \beta > 0$. A production cost is given by $c_i(q_i) = \gamma_i.q_i$, with $\gamma_i > 0$.

Game theory is based on the fundamental assumption that players are rational [8]. A rational agent takes the actions that will be most profitable for her, and to do this he has constant access to relevant information and knowledge, as well as unlimited processing capacity. The agent is aware of the available options and has clear preferences. A more detailed discussion of the notions of rationality is provided in Section 5.3, Chapter 5. We now seek to determine a solution to a normal-form game, i.e., to determine the optimal decisions that a rational player should make to minimize her costs, while taking into account the choices made by other players.

The central solution concept of noncooperative games is the notorious Nash Equilibrium (NE) defined in 1950 by the mathematician John Nash [101].

Definition 3.12. Let $G = (\mathcal{N}, \mathcal{S}, u)$. A strategy profile $s^* \in \mathcal{S}$ is a Nash equilibrium (NE) if and only if

$$\forall i \in \mathcal{N}\ \forall s_i' \in \mathcal{S}_i,\ u_i(s_i^*, s_{-i}^*) \leqslant u_i(s_i', s_{-i}^*). \tag{3.13}$$

Hence, a NE of the game is a feasible strategy profile such that no player can benefit from a unilateral deviation from his current strategy. Nash equilibrium captures the notion of a stable solution, from which no player can decrease her cost function by changing strategy. Therefore, a NE may not lead to optimal costs. In the prisoner's dilemma (Example 3.2), the best outcome for the criminals is that neither confesses (S, S). However, each player has an incentive to betray his companion, so the unique NE of the game is (C, C). Note that Definition 3.12 is expressed in case of minimization. Thereby, the inequality is reversed if rational players maximize their payoff.

A Nash equilibrium can also be defined as a strategy profile for which every player's strategy is a best response to the other players' strategies.

Definition 3.13. Let $G = (\mathcal{N}, \mathcal{S}, u)$ a strategic game. We denote the set of player i's best responses (BR) against $s_{-i} \in \mathcal{S}_{-i}$:

$$\mathcal{B}_i(s_{-i}) := \left\{ s_i \in \mathcal{S}_i \;\middle|\; u_i(s_i, s_{-i}) \leqslant u_i(s_i', s_{-i})\ \forall s_i' \in \mathcal{S}_i \right\}. \tag{3.14}$$

The best response set-valued function of the game is the application $\mathcal{B} : \mathcal{S} \to 2^{\mathcal{S}}$ defined by $\mathcal{B}(s) := \prod_{i=1}^{N} \mathcal{B}_i(s_{-i})$ for all $s \in \mathcal{S}$.

The notation 2^X stands for the set of all subsets of X. A useful way to see a NE is as a fixed point of the best response mapping for each player. Thus, a strategy profile s^* is a NE if and only if it is a fixed point of $\mathcal{B}$, i.e., if and only if $s^* \in \mathcal{B}(s^*)$.

Nash equilibria search is usually a tedious task, and not every normal-form game has one, see Example 3.5.

Example 3.5 (Matching Pennies)**.** Two players independently choose either Head or Tail. Player 1 pays player 2 one dollar if their choices differ, otherwise player 2 pays the first player one dollar. The Matching pennies game is shown in Table 3.3. The game has no Nash equilibrium.

		Player 2: Head	Player 2: Tail
Player 1	Head	$(1, -1)$	$(-1, 1)$
	Tail	$(-1, 1)$	$(1, -1)$

Table 3.3.: Matching pennies.

Therefore, we are interested in the assumptions necessary to ensure the existence of a NE. It is interesting to formulate energy exchange scheduling as a noncooperative game and find a Nash equilibrium. In this context, the strategic games used throughout multiple chapters of this thesis are continuous. So, we present an existence result for the continuous game family.

Theorem 3.3 (Debreu, Glicksberg, Fan)**.** *Let* $G = (\mathcal{N}, \mathcal{S}, u)$ *a strategic game such that for each* $i \in \mathcal{N}$

- $\mathcal{S}_i$ *is a nonempty compact convex subset of a Euclidean space,*
- u_i *is continuous in* $s \in \mathcal{S}$ *and quasi-convex in* $s_i \in \mathcal{S}_i$*.*

Then G *possesses a Nash equilibrium.*

The proof is based on Kakutani's fixed-point theorem [102]. Note that if rational players maximize their payoff function, then Theorem 3.3 requires that the b_i functions must be quasi-concave. This result guarantees the existence of at least one NE in a strategic game. Hence, we can consider the properties of the equilibria in this game, without finding them explicitly and without taking the risk of studying the empty set.

Uniqueness of NE is a desirable property, but often rare in practice due to the complexity of strategic interactions and the multitude of player preferences.

Example 3.3 (continued)**.** The NEs of the BoS game are (Movie, Movie) and (Football, Football). Note that this game is an example of a coordination game.

3.2.2. Nash equilibrium problem

Researching the Nash equilibria of a problem formulated as a normal-form game is an interesting target in itself. At this stage, we do not know how to solve this problem in a general way. However, it lies at the center of the issues that concern this thesis, particularly in the context of the day-ahead energy resources scheduling inside a REC. This framework may give rise to strategic interactions between community members, who compete for common resources (e.g., the grid) to minimize their bill. These interactions play a crucial role in collective behavior, although centralized approaches like global optimization fail to capture them. In such situations, it can be relevant to model the problem as a *Nash equilibrium problem.*

In a Nash Equilibrium Problem (NEP), each selfish player $i \in \mathcal{N}$ competes against each other by choosing her strategy $x_i \in \Omega_i \subseteq \mathbb{R}^{n_i}$ in order to minimize her objective function $b_i : \Omega = \Omega_1 \times \ldots \times \Omega_N \to \mathbb{R}$, which depends itself on other players' strategies $x_{-i} := (x_j)_{j \in \mathcal{N} \setminus \{i\}}$.

Definition 3.14. The Nash equilibrium problem (NEP) is a normal-form game $G = (\mathcal{N}, \Omega, (b_i)_{i \in \mathcal{N}})$ in which each player $i \in \mathcal{N}$ solves the following optimization problem, given $x_{-i} \in \Omega_{-i}$:

$$G := \begin{cases} \min\limits_{x_i} & b_i(x_i, x_{-i}) \qquad \forall i \in \mathcal{N} \\ \text{s.t.} & x_i \in \Omega_i \end{cases} \tag{3.15}$$

where $\Omega_i \subseteq \mathbb{R}^{n_i}$ is the strategy set constituted by the player i's individual constraints. The n-dimensional joint strategy set is expressed as $\Omega := \prod_{i \in \mathcal{N}} \Omega_i$, with $n := \sum_{i \in \mathcal{N}} n_i$.

A NEP can be seen as a set of coupled optimization problems. Note that interactions take place only at the level of the players' objective functions.

A solution of the game is a Nash equilibrium (NE), which is a feasible strategy profile such that no single player can benefit by unilaterally deviating from her strategy. The set of NEs of the game G is denoted NE(G).

Example 3.4 (continued)**.** The NEP of the Cournot duopoly is modeled as

$$\begin{aligned} &\max_{q_i} \; b_i(q_i, q_{-i}) = q_i(\alpha - \beta(q_1 + q_2)) - \gamma_i q_i \\ &\text{s.t. } \; q_i \geqslant 0. \end{aligned}$$

A Nash equilibrium $q^* = (q_1^*, q_2^*)$ is obtained by representing the best response mapping of the two producers. The profile strategy q^* is a point for which q_1^* is producer 1's best response to q_2^*, and q_2^* is one for producer 2 to q_1^*. The set of points at which the best response functions intersect is, in fact, the NEs set. In this case, there is only one intersection point and therefore, a unique NE given by

$$q^* = \Big(\frac{\alpha - 2\gamma_1 + \gamma_2}{3\beta}, \frac{\alpha - 2\gamma_2 + \gamma_1}{3\beta}\Big).$$

As a reminder, a Nash equilibrium is a fixed point of the best response mapping $\mathcal{B}$ in Definition 3.13, the fixed-point approach is therefore a standard method for the study of NEPs. However, the applicability of this analysis is strongly limited, as it may be difficult to have a closed form calculation of the best response mapping. To overcome this limitation, we can study NEPs through two alternatives, depending on the properties of the problem. These alternatives are further detailed in the subsequent sections of this chapter. First, through the reduction of a NEP to a *variational inequality problem*. The advantage of this approach lies in the well-developed variational inequality theory, allowing easy derivation of numerous results concerning solution analysis (see Section 3.3.1) and implementable solution algorithms and their convergence properties (see Section 3.4 and [99]). The second approach takes advantage of the particular structures that some games hold, such as *potential games* (see Section 3.3.2) or supermodular games [103] (which are not covered in this work).

3.2.3. Equilibrium efficiency

It is well known that in a NEP, the supposedly rational players make their decisions individually to optimize their own objectives. The selfish behavior of the players means that a NE outcome does not necessarily correspond to an optimal situation for the whole system according to a given evaluation criterion. A relevant indicator to assess the overall performance of a game (or a system) is the Social Cost (SC).

Definition 3.15. The *social cost* is the sum of the cost functions: $\mathrm{SC}(x) := \sum_{i\in\mathcal{N}} b_i(x)$, for all $x \in \Omega$.

A strategy profile $x \in \Omega$ is a social optimum if the social cost is optimized. A fundamental issue is to evaluate the extent to which individual decisions at Nash equilibria lead to outcomes that deviate from the socially optimal solution. The socially optimal solution is obtained through global optimization, where a central operator coordinates all players' actions to minimize the social cost.

A classic quantitative measure of the efficiency of Nash equilibria is the Price of Anarchy (PoA). It is defined as the ratio between the worst value attainable by the social cost at an equilibrium and the optimal social cost.

Definition 3.16 ([18])**.** Given a game G and NE(G) its set of Nash equilibria, *the price of anarchy* (PoA) of G is defined by

$$\text{PoA}(G) := \frac{\max_{x^* \in \text{NE}(G)} \text{SC}(x^*)}{\min_{x \in \Omega} \text{SC}(x)}. \tag{3.16}$$

We illustrate this concept with the example of the prisoner's dilemma presented earlier.

Example 3.2 (continued)**.** As a reminder, the unique Nash equilibrium of the prisoner's dilemma is (C, C), which induces SC=2+2=4. The optimal social cost value is 2 for this game. Thus, PoA=4/2=2 and the NE is inefficient.

The PoA measures the distance between the highest social cost observed at a Nash equilibrium and the socially optimal solution. Intuitively, this ratio quantifies the worst-case efficiency loss, defined as the increase in social cost resulting from the decentralized decision-making of selfish agents, compared to the optimal solution that could be achieved through centralized approaches. For a game with multiple equilibria, a price of anarchy close to 1 indicates that all its equilibria are good approximations of a social optimum outcome. Conversely, a PoA greater than 1 signifies that at least one NE results in a social cost higher than the optimal solution. However, a larger PoA does not allow us to conclude that all equilibria are inefficient [104].

Another measure of inefficiency differentiates games where some NEs are inefficient from games where all equilibria are inefficient. The Price of Stability (PoS) of a game is given by the ratio between the best social cost value observed at an equilibrium and the optimal social cost.

Definition 3.17 ([105])**.** Given a game G and NE(G) its set of Nash equilibria, *the price of stability* (PoS) of G is defined by

$$\text{PoS}(G) := \frac{\min_{x^* \in \text{NE}(G)} \text{SC}(x^*)}{\min_{x \in \Omega} \text{SC}(x)}. \tag{3.17}$$

If a game has a unique equilibrium, the PoA and PoS are identical (see Examples 3.2 and 3.4).

A price of stability equal to 1 indicates that at least one NE is a social optimum, meaning that there exists an outcome in the game that achieved the optimal solution obtained through centralized optimization. On the other hand, a PoS greater than 1 implies that even the best social cost observed at a NE is still higher than the socially optimal solution. It is easy to see from the definitions that the PoS of a game G is at least as close to 1 as its PoA,

$$1 \leqslant \mathrm{PoS}(G) \leqslant \mathrm{PoA}(G).$$

Note that Definitions 3.16 and 3.17 are expressed in case of minimization. In case of maximization, the definitions are turned over (see Example 3.6)

Example 3.6. We take the BoS game shown in Example 3.3. Our couple has finally agreed to go to the cinema, but now they are arguing about the movie choice between A and B. Again, there are two NEs: (A, A) with values 4 and (B, B) with values 12. The optimal value is 12. Then, PoS=12/12=1 and PoA=12/4=3. The equilibrium (B, B) is a social optimum, while (A, A) is inefficient.

		Eric	
		A	B
Caroline	A	$(3,1)$	$(0,0)$
	B	$(0,0)$	$(2,10)$

Table 3.4.: Modified Battle of Sexes game.

3.2.4. Generalized Nash equilibrium problem

The Nash equilibrium problem framework assumes that each players' decisions affect the other players only through their individual objective functions. However, in many situations, strategic interactions can also affect players' decisions through constraints, whether individual or shared at the system level. This therefore necessitates a model enabling a more accurate representation of systems where players are interdependent not only through their objectives but also via their strategy sets. For instance, power system stability and reliability depend on capital power balanced equality constraints. These constraints are directly influenced by the variables of energy market participants, such as generators' production capacity, commodity demand and the operational conditions of transmission infrastructures. This complex interdependence of constraints requires a framework in which the interactions between players are explicitly incorporated into the system constraints in addition to the objective functions of the participants.

The Generalized Nash Equilibrium Problem (GNEP) extends the classical NEP by assuming that each player's feasible set can depend on the rival players' strategies. Hence, the feasible sets are not fixed, as it is the case in classical game theory. Each player $i \in \mathcal{N}$ has a point-to-set map $\Omega_i : \prod_{j \neq i} \mathbb{R}^{n_j} \to 2^{\mathbb{R}^{n_i}}$.

Definition 3.18. The generalized Nash equilibrium problem (GNEP) $\mathcal{G} = (\mathcal{N}, (\Omega_i)_{i \in \mathcal{N}}, (b_i)_{i \in \mathcal{N}})$ can be formally defined as a problem in which each player $i \in \mathcal{N}$ simultaneously solves the following optimization problem, given other players' strategies x_{-i}:

$$\mathcal{G} := \begin{cases} \min\limits_{x_i} & b_i(x_i, x_{-i}) \qquad \forall i \in \mathcal{N} \\ \text{s.t.} & x_i \in \Omega_i(x_{-i}) \end{cases} \tag{3.18}$$

where the strategy of player i must belong to the feasible set $\Omega_i(x_{-i}) \subseteq \mathbb{R}^{n_i}$ that depends on the other players' strategies. The point-to-set mapping $\Omega : \mathbb{R}^n \to 2^{\mathbb{R}^n}$ is defined as $\Omega(x) := \prod_{i \in \mathcal{N}} \Omega_i(x_{-i})$, for all $x \in \mathbb{R}^n$ and $n := \sum_{i \in \mathcal{N}} n_i$.

Example 3.7. Consider the energy production duopoly (Example 3.4). We note q_{max} the maximum total production for the market based on common constraints. The new problem for producer i is

$$\begin{aligned} \max_{q_i}\ & q_i.(\alpha - \beta(q_1 + q_2)) - \gamma_i q_i \\ \text{s.t.}\ & q_i \geqslant 0 \\ & q_1 + q_2 \leqslant q_{max}. \end{aligned}$$

A solution of a GNEP is called a *generalized Nash equilibrium.*

Definition 3.19. A strategy profile x^* is called a Generalized Nash Equilibrium (GNE) of the game $\mathcal{G}$ (3.18), if for all $i \in \mathcal{N}$:

$$b_i(x_i^*, x_{-i}^*) \leqslant b_i(x_i, x_{-i}^*), \quad \forall x_i \in \Omega_i(x_{-i}^*). \tag{3.19}$$

The set of generalized Nash equilibria of $\mathcal{G}$ is noted GNE($\mathcal{G}$).

A GNE is a feasible strategy profile such that no single player can benefit by unilaterally deviating from her strategy. So, the idea is the same as the NEs, but applies to all $x_i \in \Omega_i(x_{-i}^*)$ in the case GNEs.

We assume that for each player $i \in \mathcal{N}$, the strategy set is defined explicitly by inequality and equality constraints, such as:

$$\Omega_i(x_{-i}) := \left\{ y_i \in \overline{\Omega}_i \;\middle|\; g_i(y_i, x_{-i}) \leqslant 0,\ h_i(y_i, x_{-i}) = 0 \right\} \tag{3.20}$$

where $\overline{\Omega}_i \subseteq \mathbb{R}^{n_i}$ is the player i's individual constraints set, $g_i(\cdot, x_{-i}) : \mathbb{R}^{n_i} \to \mathbb{R}^{m_i}$ and $h_i(\cdot, x_{-i}) : \mathbb{R}^{n_i} \to \mathbb{R}^{p_i}$ are respectively the set of inequality and equality constraints functions of player i that also depend on the other players' strategies. For every player i, $g_i := (g_{i,j})_{j=1}^{m_i}$ and $h_i := (h_{i,k})_{k=1}^{p_i}$. Note that if $\Omega_i(x_{-i}) = \overline{\Omega}_i$ for all player $i \in \mathcal{N}$, the feasible sets do not depend on the other players' variables, so the GNEP reduces to the standard NEP (3.15).

Some notations, a function of class C^k is a function that has a k-th derivative that is continuous in its domain. The class C^0 consists of all continuous functions. We call a function of class C^1 continuously differentiable.

This thesis considers a special class of GNEPs for which a much more complete theory exists in the literature [106, 107].

Definition 3.20. Let a GNEP $\mathcal{G}$, suppose that for every player $i \in \mathcal{N}$ the objective function is C^0 and for every x_{-i}, $b_i(\cdot, x_{-i})$ is convex and the set $\Omega_i(x_{-i})$ is closed and convex. This GNEP is jointly convex if for some closed convex set $\mathcal{C} \subseteq \mathbb{R}^n$ and all $i \in \mathcal{N}$, we have

$$\Omega_i(x_{-i}) = \{x_i \in \mathbb{R}^{n_i} \mid (x_i, x_{-i}) \in \mathcal{C}\}. \tag{3.21}$$

When strategy sets are defined as in (3.20), then (3.21) is equivalent to the requirement that $g := g_1 = \ldots = g_N$ and $h := h_1 = \ldots = h_N$, such as

$$\mathcal{C} = \left\{x \in \mathbb{R}^n \mid x_i \in \overline{\Omega}_i \ \forall i \in \mathcal{N}, \ g(x) \leqslant 0, \ h(x) = 0\right\}, \tag{3.22}$$

where $\overline{\Omega}_i$ is closed and convex for each player i, $g(x)$ are (componentwise) convex with respect to all variables x and $h(x)$ are affine. In fact, g and h represent the sets of shared coupling constraints, i.e. inequality and equality constraints that are equal for all the players. Jointly convex GNEPs are also termed as GNEPs with shared constraints or coupled constraints [107].

Example 3.7 (continued). For each firm $i \in \{1, 2\}$, we have b_i continuously differentiable in $q = (q_1, q_2)$. The function $-b_i(\cdot, q_{-i})$ is convex for every q_{-i}. The individual strategy sets $\overline{\Omega}_i = \mathbb{R}_+$ are closed and convex, furthermore, the function $g(q_i, q_{-i}) = q_1 + q_2 - q_{max}$ is linear and, therefore, convex on q. We have $\Omega_i(q_{-i}) = \{q_i \in \mathbb{R}_+ \mid g(q_i, q_{-i}) \leqslant 0\}$ closed and convex, which can be given as (3.21) with

$$\mathcal{C} = \left\{q \in \mathbb{R}^2 \mid q_1 \in \mathbb{R}_+, q_2 \in \mathbb{R}_+, q_1 + q_2 - q_{max} \leqslant 0\right\}$$

closed and convex too. Then, the GNEP is jointly convex by Definition 3.20.

Similar to NEs, the outcomes of GNEs represent a stable situation, but not necessarily optimal for the system as a whole. This sub-optimality highlights the potential inefficiencies inherent in decentralized decision-making models. To quantify these inefficiencies, the definitions of price of anarchy (PoA) and the price of stability (PoS), previously introduced in Definitions 3.16 and 3.17 for NEP, can be extended to the GNEP framework.

At this stage, we consider that the reader has all the information needed to understand the various model formulations in Chapter 4. Table 3.5 provides a summary of the different conceptual tools. The next sections introduce mathematical and algorithmic tools for analyzing and solving the problems presented so far. Thus, readers less interested in these theoretical technical aspects can skip ahead to the next chapter.

Optimization Problem	Nash Equilibrium Problem	Generalized Nash Equilibrium Problem
$\min_{x} \ f(x)$ s.t. $x \in \Omega$	$\min_{x_i} \ b_i(x_i, x_{-i})$ s.t. $x_i \in \Omega_i$	$\min_{x_i} \ b_i(x_i, x_{-i})$ s.t. $x_i \in \Omega_i(x_{-i})$
One decision-maker. One objective function to be minimized. A feasible set.	Several decision-makers. Each player minimizes her own cost function. Cost functions depend on the other players' variables. Strategy sets are independent of the other players' choices.	Several decision-makers. Each player minimizes her own cost function. Cost functions depend on the other players' variables. Strategy sets depend on the other players' variables.
Continuous problems & simultaneous decisions		

Table 3.5.: Comparative summary of problems addressed: *optimization problem*, *Nash equilibrium problem* and *generalized Nash equilibrium problem.*

3.3. Mathematical tools to solve (G)NEPs

3.3.1. Variational inequality theory

Variational Inequality (VI) theory provides a general framework that integrates a broad range of mathematical problems, applied sciences and economic [99, 100]. This section presents some theoretical foundations in finite-dimensional variational inequalities and how convex optimization problems and (generalized) Nash equilibrium problems can be formulated as a VI problem.

Variational inequality problem

A variational inequality problem is defined as follows.

Definition 3.21. Given a set $\mathcal{K} \subseteq \mathbb{R}^n$ and a vector-valued mapping $F : \mathcal{K} \to \mathbb{R}^n$. The *variational inequality problem*, denoted by $\mathrm{VI}(\mathcal{K}, F)$, is to determine a vector $x^* \in \mathcal{K}$, such that

$$(y - x^*)^\top F(x^*) \geqslant 0, \ \forall y \in \mathcal{K}. \tag{3.23}$$

The solutions set of $\mathrm{VI}(\mathcal{K}, F)$ is noted $\mathrm{SOL}(\mathcal{K}, F)$.

The simplest example of a variational inequality is the classical problem of solving a system of nonlinear equations. If $\mathcal{K} = \mathbb{R}^n$, the only vector $F(x^*)$ which forms a non-obtuse angle with all vectors in $\mathbb{R}^n$ is the zero vector. Then $\mathrm{VI}(\mathbb{R}^n, F)$ is equivalent to finding a $x^* \in \mathbb{R}^n$ such that $F(x^*) = 0$

For the sake of simplicity, we assume that F is continuously differentiable. If $F = \nabla f$ for some suitable convex function f on the convex set $\mathcal{K}$, then the $\mathrm{VI}(\mathcal{K}, \nabla f)$ corresponds to the problem of finding points satisfying the minimum principle (3.10) and so with the resolution of the convex optimization problem (3.8). Figure 3.4 illustrates a geometrical interpretation of (3.23).

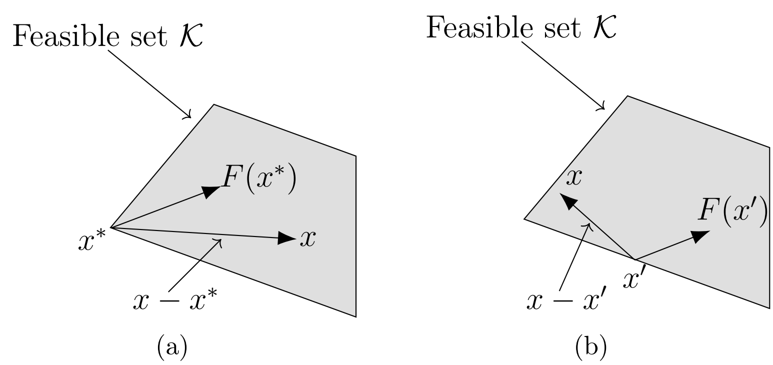


Figure 3.4.: Geometric interpretation of VIs: (a) a feasible point x^* that is a solution of the $\mathrm{VI}(\mathcal{K}, F)$ as all feasible vector $x - x^*$ form an acute angle with $F(x^*)$; and (b) a feasible point x' that is not a solution of $\mathrm{VI}(\mathcal{K}, F)$.

However, not all the VI problems have the property that the vector-valued mapping F is a gradient map of a function [100]. The equivalence between optimization and the VI is given by the principle of symmetry.

Theorem 3.4 ([99]). *Let $F : \mathcal{U} \to \mathbb{R}^n$ be continuously differentiable on the open convex set $\mathcal{U} \subseteq \mathbb{R}^n$. The following three statements are equivalent:*

1. *there exists a real-valued function f such that $F(x) = \nabla f(x)$ for all $x \in \mathcal{U}$;*
2. *the Jacobian matrix $JF(x)$ is symmetric for all $x \in \mathcal{U}$;*
3. *F is integrable on $\mathcal{U}$.*

If any one of these statements holds, then the function f can be given by

$$f(x) := \int_0^1 F(x^0 + t(x - x^0))^\top (x - x^0) dt$$

where $x^0 \in \mathcal{U}$ is an arbitrary vector.

Recall that a matrix A is symmetric if $A^\top = A$. Furthermore, the Jacobian matrix of F at a point is defined as follows.

Definition 3.22. Let $a \in \mathbb{R}^n$ and $F : \mathbb{R}^n \to \mathbb{R}^m$ differentiable in a, the Jacobian matrix of the function F at the point a is

$$JF(a) := \begin{pmatrix} \frac{\partial F_1}{\partial x_1}(a) & \dots & \frac{\partial F_1}{\partial x_n}(a) \\ \vdots & \ddots & \vdots \\ \frac{\partial F_m}{\partial x_1}(a) & \dots & \frac{\partial F_m}{\partial x_n}(a) \end{pmatrix}.$$

We present a first result about the existence and structure of the solution set of a VI problem. It can be considered as the natural extension of Weierstrass theorem for optimizations problems [108].

Theorem 3.5. *Given a VI($\mathcal{K}, F$), suppose that*

1. *The set $\mathcal{K}$ is convex and compact (closed and bounded).*
2. *The function F is continuous.*

Then, the set of solutions $\mathrm{SOL}(\mathcal{K}, F)$ is nonempty and compact.

The boundedness assumption of the set might be too restrictive in the Theorem 3.5. We can do without this assumption under certain additional properties of the function F. For this purpose, we provide some function classes.

Definition 3.23. Let $E \subseteq \mathbb{R}^n$ a closed and convex set, a mapping $F : E \to \mathbb{R}^n$ is said to be

- monotone on E if, for all $x, y \in E$

$$(x - y)^\top (F(x) - F(y)) \geqslant 0 \tag{3.24}$$

- strictly monotone on E if, for all $x, y \in E$ such as $x \neq y$

$$(x - y)^\top (F(x) - F(y)) > 0 \tag{3.25}$$

- strongly monotone on E with parameter $\tau > 0$ if, for all $x, y \in E$

$$(x - y)^\top (F(x) - F(y)) \geqslant \tau \|x - y\|^2. \tag{3.26}$$

We say that the VI($\mathcal{K}, F$) is monotone if the mapping function F is monotone. We do the same for the other properties. Note that if F is an affine function, then the strict monotonicity is the same as strong monotonicity.

From Definitions 3.23, we can state that the following relation holds.

Let $E \subseteq \mathbb{R}^n$ a closed and convex set, and a mapping $F : E \to \mathbb{R}^n$, we have

$$F \text{ strongly monotone} \Rightarrow F \text{ strictly monotone} \Rightarrow F \text{ monotone.}$$

Note that if the function F is the gradient of a differentiable function f then the monotony property of F can be related to the convexity of f [108].

Thanks to the monotonicity properties, we can state the following theorem without requiring the boundedness of the set $\mathcal{K}$.

Theorem 3.6. *Given a VI($\mathcal{K}, F$), suppose that $\mathcal{K}$ is closed and convex and F is continuous on $\mathcal{K}$. The following statements hold:*

1. *If F is monotone on $\mathcal{K}$, then the VI($\mathcal{K}, F$) has a (possible empty) convex solution set,*
2. *If F is strictly monotone on $\mathcal{K}$, then the VI($\mathcal{K}, F$) has at most one solution,*
3. *If F is strongly monotone on $\mathcal{K}$, then the VI($\mathcal{K}, F$) has a unique solution.*

Note that the strict monotonicity of F does not guarantee the existence of a solution. Further existence and uniqueness results are given in [99], for certain classes of functions.

VI formulation of NEP

We have seen that there are at least two other ways of studying the properties of Nash equilibrium problems (3.2.2). The first approach is to reformulate the NEP as a suitable variational inequality problem in order to exploit the well-developed VI theory. It provides a rich framework for analyzing solutions, developing practical algorithms and studying their convergence properties.

We present the proposition establishing the equivalence between a NEP and an appropriate VI, under certain conditions. We assume that the equivalence between two models means that the solutions of one correspond exactly to the solutions of the other. Then, the initial problem can be solved or analyzed using an alternative formulation, while ensuring that the fundamental properties and results obtained remain identical in both frameworks.

Proposition 3.1. *Given a NEP $G = (\mathcal{N}, (\Omega_i)_{i\in\mathcal{N}}, (b_i)_{i\in\mathcal{N}})$, suppose that for each player i:*

1. *the strategy set $\Omega_i \subseteq \mathbb{R}^{n_i}$ is closed and convex,*
2. *the payoff function b_i is continuously differentiable in x and convex in x_i for every fixed $x_{-i} \in \Omega_{-i}$.*

Then, the NEP G is equivalent to the VI(Ω, F), i.e., $\mathrm{NE}(G) = \mathrm{SOL}(\Omega, F)$ *where*

$$\Omega := \prod_{i=1}^{N} \Omega_i \qquad\qquad F(x) := \begin{pmatrix} \nabla_{x_1} f_1(x) \\ \vdots \\ \nabla_{x_N} f_N(x) \end{pmatrix}.$$

Actually, this connection naturally arises from the minimum principle for convex problems (Definition 3.10) and the Cartesian structure of the combined strategy set Ω [99]. We take the example of the Cournot duopoly to illustrate this result.

Example 3.8. In the Cournot duopoly (Example 3.4), the energy producers' feasible sets correspond to the nonnegative real number set $\mathbb{R}_+$ which is closed and convex. Furthermore, $-b_i$ is continuously differentiable and convex for both players. Then, the Proposition 3.1 provides the equivalent VI problem VI(Ω, F) where $\Omega = \mathbb{R}_+ \times \mathbb{R}_+$ and

$$F(q_1, q_2) := \begin{pmatrix} \gamma_1 + 2\beta q_1 + \beta q_2 - \alpha \\ \gamma_2 + 2\beta q_2 + \beta q_1 - \alpha \end{pmatrix}.$$

Furthermore, we can easily show with the Definition 3.23 that F is strongly monotone with parameter $\beta > 0$. As a result, Theorem 3.6 shows that the VI problem has a unique solution.

VI formulation of GNEP

We have shown the connection between NEPs and VI problems through the Proposition 3.1. We would like to obtain a result of the same order in the case where the feasible sets are dependent on the other players' strategies, i.e., GNEP (3.2.4). However, it is not as immediate as in the NEPs case. A GNEP can be formulated as a *Quasi-Variational Inequality* (QVI). The QVIs are a generalization of VI problems, in which the defining set of the problem varies with the variables.

Definition 3.24. Given a point-to-set mapping $\mathcal{K} : \mathbb{R}^n \to \mathcal{P}(\mathbb{R}^n)$ and a mapping $F : \mathbb{R}^n \to \mathbb{R}^n$. The quasi-variational inequality problem, denoted by QVI($\mathcal{K}$, F), is to determine a vector $x^* \in \mathcal{K}(x^*)$, such that :

$$(y - x^*)^\top F(x^*) \geqslant 0, \quad \forall y \in \mathcal{K}(x^*).$$

Hence, if the point-to-set mapping $\mathcal{K}$ is constant, then the QVI problem reduces to a VI problem.

Given some conditions, it is possible to establish the link between the GNEP and a QVI problem [109, 110].

Proposition 3.2. *Given a GNEP $\mathcal{G}$, suppose that for each player i:*

1. *the payoff function b_i is continuously differentiable in x,*
2. *for every x_{-i}, the function $b_i(\cdot, x_{-i})$ is convex in x_i and the set $\Omega_i(x_{-i})$ is closed and convex.*

Then, the GNEP $\mathcal{G}$ is equivalent to the QVI(Ω, F), where the point-to-set mapping $\Omega : \mathbb{R}^n \to 2^{\mathbb{R}^n}$ is defined $\Omega(x) := \prod_{i\in\mathcal{N}} \Omega_i(x_{-i})$ and $F(x) := (\nabla_{x_i} b_i(x))_{i\in\mathcal{N}}$ for all $x \in \mathbb{R}^n$.

Although this result is theoretically valid, it should be noted that, in practice, methods for solving such a generic problem are still very limited and not necessarily accessible in real-life cases.

Nevertheless, this thesis considers a special class of equilibrium problems: the jointly convex GNEPs (3.20), for which meaningful results can be obtained. Though jointly convex GNEPs remain complex problems, they can be solved by finding the solution to a suitable VI problem [111, 106].

Proposition 3.3. *Given a GNEP $\mathcal{G}$, suppose that for each player i:*

1. *the payoff function b_i is continuously differentiable in x,*
2. *for every x_{-i}, the function $b_i(\cdot, x_{-i})$ is convex in x_i,*
3. *the sets $\Omega_i(x_{-i})$ are defined by* (3.21) *with $\mathcal{C}$ closed and convex.*

Then, each solution of the VI($\mathcal{C}, F = (\nabla_{x_i} b_i)_{i\in\mathcal{N}}$) *is a solution of the GNEP.*

Proposition 3.3 does not state that any solution of the GNEP is also a solution of the associated VI problem. In fact, not all the GNEs are preserved in passing from GNEP to VI. Therefore, it could happen that the GNEP has a solution, but VI has none.

Example 3.9. Consider the GNEP $\mathcal{G}_1$ with two players

$$\begin{aligned} &\min_x \ (x-1)^2 & \qquad & \min_y \ \left(y-\frac{1}{2}\right)^2 \\ &\text{s.t. } x+y \leqslant 1 & & \text{s.t. } x+y \leqslant 1. \end{aligned}$$

It can be shown that this game has infinitely many GNEs given by $(\alpha, 1-\alpha)$ for every $\alpha \in [1/2, 1]$. The VI associated VI(C, F)

$$C = \{(x,y) \in \mathbb{R}^2 : x+y \leqslant 1\} \qquad F(x,y) = (2x-2 \ \ 2y-1)^\top$$

with F strongly monotone, so the VI has a unique solution SOL$(C,F) = \{(3/4, 1/4)\}$. Then GNE$(\mathcal{G}_1) \nsubseteq$ SOL(C,F).

Generalized Nash equilibria of a GNEP, which also satisfy the associated VI problem, are called variational equilibria. They represent a subset of GNEP solutions.

Definition 3.25. Let a jointly convex GNEP $\mathcal{G}$. A solution of the GNEP $\mathcal{G}$ that is also a solution of VI($\mathcal{C}, F$) is a Variational Equilibrium (VE).

Hence, thanks to the VI formulation, the VEs of a GNEP can be studied and calculated more easily, relying on the much more developed VI theory. In Example 3.9, the point $(3/4, 1/4)$ is the unique variational equilibrium of the game.

Since not every GNE satisfies the VI, it is pertinent to consider the special characteristics that distinguish the variational equilibria. To do this, we need to introduce the KKT conditions for both GNEP and VI problems.
We observe a GNEP $\mathcal{G}$ defined by (3.18) and for each player $i \in \mathcal{N}$, the strategy set is given by (3.20):

$$\Omega_i(x_{-i}) := \left\{ y_i \in \overline{\Omega}_i \;\middle|\; g_i(y_i, x_{-i}) \leqslant 0, \ h_i(y_i, x_{-i}) = 0 \right\}.$$

We assume that all objective functions, inequality functions and equality functions involved are C^1. A Nash equilibrium problem (generalized or classical) constituted interrelated optimization problems. In this way, we can easily derive the KKT optimal conditions for each player's optimization problem and then, we can deduce from all these conditions what we can call the KKT conditions of the GNEP, as illustrated in Fig.3.5.

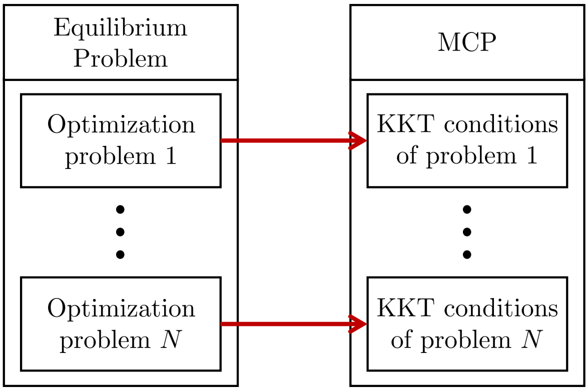


Figure 3.5.: KKT conditions of an equilibrium problem: joint solution of several systems of KKT conditions.

We now translate this reasoning into mathematical terms. For the sake of clarity, we disregard the individual constraint sets $\overline{\Omega}_i$. The classical KKT conditions of player i's optimization problem is given by:

$$\begin{aligned}
&\nabla_{x_i} L_i(x, \lambda_i, \mu_i) := \nabla_{x_i} b_i(x) + \sum_{j=1}^{m_i} \lambda_{i,j} \nabla_{x_i} g_{i,j}(x) + \sum_{k=1}^{p_i} \mu_{i,k} \nabla_{x_i} h_{i,k}(x) = 0 \\
&0 \leqslant \lambda_i \perp g_i(x) \leqslant 0 \\
&h_{i,k}(x) = 0, \ \forall k \in \{1, \dots, p_i\}
\end{aligned} \tag{3.27}$$

where $\lambda_i \in \mathbb{R}^{m_i}$ and $\mu_i \in \mathbb{R}^{p_i}$ are the Lagrange multipliers. The concatenation of these N-KKT conditions provides the following system

$$\begin{aligned}
&\mathbf{L}(x, \lambda, \mu) = 0 \\
&0 \leqslant \lambda \perp \mathbf{G}(x) \leqslant 0 \\
&\mathbf{H}(x) = 0
\end{aligned} \tag{3.28}$$

where $\lambda := (\lambda_i)_{i=1}^N \in \mathbb{R}^{\mathbf{m}}$ with $\mathbf{m} := \sum_{i=1}^N m_i$, $\mu := (\mu_i)_{i=1}^N \in \mathbb{R}^{\mathbf{p}}$ with $\mathbf{p} := \sum_{i=1}^N p_i$, $\mathbf{G} := (g_i)_{i=1}^N$, $\mathbf{H} := (h_i)_{i=1}^N$ and $\mathbf{L} := (\nabla_{x_i} L_i)_{i=1}^N$.
Similar to the Theorem 3.2, we can provide a result that establishes a clear

connection between a GNEP and its KKT conditions.

Theorem 3.7. *Let a GNEP* $\mathcal{G} = (\mathcal{N}, (\Omega_i)_{i\in\mathcal{N}}, (b_i)_{i\in\mathcal{N}})$ (3.18)*, with the strategy sets defined by* (3.20)*. All objective functions, inequality functions and equality functions involved are* C^1*.*

- *Let* x^* *be a solution of the GNEP at which all the players' subproblems satisfy a constraint qualification [112]. Then there exist Lagrange multipliers* λ^* *and* μ^* *such that* (x^*, λ^*, μ^*) *solves system* (3.28)*.*
- *If* (x^*, λ^*, μ^*) *solves the system* (3.28) *and that for every player* $i \in \mathcal{N}$ *and every* x_{-i}*, the function* $b_i(\cdot, x_{-i})$ *is convex and the set* $\Omega_i(x_{-i})$ *is closed and convex. Then* x^* *is a GNE of the GNEP.*

We now discuss the implications of Theorem 3.7 within the framework of an additional hypothesis. In the case of a jointly convex GNEP with the feasible set (3.22), it follows that the system (3.28) is given with $\mathbf{G}{:=}(g)_{i=1}^N$ and $\mathbf{H}{:=}(h)_{i=1}^N$ and for each player i we have $m_i = m$ and $p_i = p$ for fixed $m, p \in \mathbb{N}$. This implies that each vector λ_i is orthogonal to the same condition g, but the Lagrange multipliers may vary between players. Next, consider the KKT conditions of the VI$(\mathcal{C}, F)$ from Proposition 3.3

$$\begin{aligned} &F(x) + \nabla_x g(x)\lambda + \nabla_x h(x)\mu = 0 \\ &0 \leqslant \lambda \perp g(x) \leqslant 0 \\ &h(x) = 0 \end{aligned} \tag{3.29}$$

where $\lambda \in \mathbb{R}^m$ and $\mu \in \mathbb{R}^p$ are the Lagrange multipliers. The relation between the VI problem and the KKT conditions are established in [99, Prop. 1.3.4].

The next result establishes the connection between the two KKT systems and a VE of the jointly convex GNEP, assuming it satisfies a constraint qualification [112].

Theorem 3.8 ([110, 111])**.** *Let a jointly convex GNEP where the feasible sets are defined by* (3.21) *with* $\mathcal{C}$ *convex given by* (3.22)*.*

- *Let* x^* *be a solution of the VI*$(\mathcal{C}, F)$ *at which* (3.29) *holds with some multipliers* $\lambda^* \in \mathbb{R}^m$ *and* $\mu^* \in \mathbb{R}^p$*. Then* x^* *is a solution of the GNEP, and the corresponding KKT conditions* (3.28) *are satisfied with* $\lambda^* := \lambda_1 := \ldots := \lambda_N$ *and* $\mu^* := \mu_1 \ldots := \mu_N$*.*
- *Let* x^* *be a solution of the GNEP such that* (3.28) *hold with* $\lambda_1^* = \ldots = \lambda_N^*$ *and* $\mu_1^* = \ldots = \mu_N^*$*. Then* x^* *is a solution of VI*$(\mathcal{C}, F)$ *and the point* (x^*, λ^*, μ^*) *with* $\lambda^* := \lambda_1^*$ *and* $\mu^* := \mu_1^*$*, satisfies* (3.29)*.*

In other words, a VE is a solution for the jointly convex GNEP if and only if the shared constraints have the same Lagrange multipliers for all the players.

This property can lead to a very interesting economic interpretation, notably in the context of energy markets, as some of these multipliers can be interpreted as shadow prices (see e.g., Appendix B.3, [66], etc.). It is therefore common practice to restrict the resolution of a jointly convex GNEP to the calculation of its VEs (e.g., [113, 66]).

A possible disadvantage is the fact that one can compute VEs only, excluding possible other solutions that might be interesting. It can be relevant to study how efficient those GNEs outcomes can be in comparison to the VE outcome [66]. Nabetani et al. [114] propose two types of parametrized VIs approaches, which allow us to evaluate the GNEs set of a jointly convex GNEPs. On the basis of these approaches, we have been able to establish theoretical results relating to the characterization of GNEs for the problems studied in Chapter 4. However, in this report, we choose to focus primarily on variational equilibria, because of the theoretical and algorithmic tools available for their analysis and resolution.

3.3.2. Potential games

In Section 3.3.1, we have shown that a Nash equilibrium problem can be studied by reducing the NEP to a VI problem. Some other approaches exploit the specific structure of certain games. In this section, we introduce a particular class of games called *potential games.* This class of game has many strong implications for the existence and convergence to equilibria.

The first concept of potential games can be found in Rosenthal's work from 1973 [115], but they were formally introduced by Monderer and Shapley in 1996 [116]. This class of games has the property that the incentive of all players to unilaterally deviate from a strategy profile can be expressed in one global function, the *potential function.* The specific relationship between the potential function and the payoff functions of players determines the classification of potential games. We list three types: ordinal potential games, weighted potential games and exact potential games, but there exist various types of potential games in the literature [104, 117].

Let $G = (\mathcal{N}, (\mathcal{S}_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$ be a strategic game with a finite number of players.

Definition 3.26. The game G is an *exact potential game* (EPG) if and only if a potential function $P : \mathcal{S} \to \mathbb{R}$ exists such that for all $i \in \mathcal{N}$ and $s_{-i} \in \mathcal{S}_{-i}$

$$u_i(x, s_{-i}) - u_i(y, s_{-i}) = P(x, s_{-i}) - P(y, s_{-i}), \ \forall x, y \in \mathcal{S}_i. \tag{3.30}$$

Definition 3.27. The game G is a *weighted potential game* (WPG) if and only if a potential function $P : \mathcal{S} \to \mathbb{R}$ exists such that for all $i \in \mathcal{N}$ and $s_{-i} \in \mathcal{S}_{-i}$

$$u_i(x, s_{-i}) - u_i(y, s_{-i}) = w_i.(P(x, s_{-i}) - P(y, s_{-i})), \ \forall x, y \in \mathcal{S}_i, \tag{3.31}$$

where $w = (w_i)_{i\in\mathcal{N}}$ is a vector of positive numbers called weights.

Obviously, an exact potential game is a weighted potential game where all the players have weights equal to one.

Definition 3.28. The game G is an *ordinal potential game* (OPG) if and only if a potential function $P : \mathcal{S} \to \mathbb{R}$ exists such that for all $i \in \mathcal{N}$ and $s_{-i} \in \mathcal{S}_{-i}$

$$u_i(x, s_{-i}) - u_i(y, s_{-i}) > 0 \Leftrightarrow P(x, s_{-i}) - P(y, s_{-i}) > 0, \ \forall x, y \in \mathcal{S}_i. \tag{3.32}$$

Intuitively, the ordinal potential game requires that the change in the potential function due to a unilateral strategy deviation needs to be of the same sign as the change in the player's payoff function. Clearly, an EPG is an OPG, but not the other way around. Through the whole thesis, a game will be called a potential game (PG) if it has a potential function.

Example 3.10. The Cournot duopoly in Example 3.4, is a potential game where the exact potential function $P : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}$ is defined as:

$$P(q_1, q_2) = \alpha(q_1 + q_2) - \beta(q_1^2 + q_2^2) - \beta q_1 q_2 - \gamma_1 q_1 - \gamma_2 q_2, \quad \forall q_1, q_2 \in \mathbb{R}_+.$$

If we assume that both generators always produce something, i.e., $q_1, q_2 \in \mathbb{R}_+^0$, then the function $P : \mathbb{R}_+^0 \times \mathbb{R}_+^0 \to \mathbb{R}$ defined as:

$$P(q_1, q_2) = q_1 q_2(\alpha - \beta(q_1^2 + q_2^2)) - \gamma_1 q_1 - \gamma_2 q_2, \quad \forall q_1, q_2 \in \mathbb{R}_+^0,$$

is an ordinal potential function.

Example 3.11. We consider a set of N tasks $\mathcal{N} = \{1, \ldots, N\}$ that need to be scheduled on one of the M identical machines available $\mathcal{M} = \{1, \ldots, M\}$. Each job $i \in \mathcal{N}$ has a weight w_i, representing the execution time of the task. A player i's strategy is to choose a machine $j \in \mathcal{M}$, thus $\mathcal{S}_i = \mathcal{M}$ and $\mathcal{S} = \mathcal{M}^N$. The load of a machine $j \in \mathcal{M}$ is defined as the sum of the tasks assigned to that machine, given the strategy profile $s \in \mathcal{S}$, $l_j(s) = \sum_{i,s_i=j} w_i$. The objective of each player is to minimize the total load of the machine on which job i is working: $c_i(j, s_{-i}) = l_j(s)$. This load balancing game is a potential game with the weighted potential function $P : \mathcal{S} \to \mathbb{R}$ defined as:

$$P(s) = \frac{1}{2} \sum_{j \in \mathcal{M}} l_j(s)^2, \quad \forall s \in \mathcal{S}.$$

We present how potential functions can help to better analyze a potential game and to find Nash equilibria. In fact, a Nash equilibrium in potential games can be established by the finding that the NEs set corresponds to that of a game where all players minimize the potential function.

Theorem 3.9 (Monderer and Shapley)**.** *If $P : \mathcal{S} \to \mathbb{R}$ is a potential function for the ordinal potential game $G = (\mathcal{N}, \mathcal{S}, (u_i)_{i\in\mathcal{N}})$, then the set of Nash equilibria of G coincides with the Nash equilibria set of the game $\tilde{G} = (\mathcal{N}, \mathcal{S}, (P)_{i\in\mathcal{N}})$:*

$$\mathrm{NE}(G) = \mathrm{NE}(\tilde{G}).$$

A consequence of this result is that the NEs properties can be studied using only the potential function. Clearly, if the potential function P has a minimum point $x^* \in \mathcal{S}$, then x^* is a Nash equilibrium for the game G. However, the converse is usually not true. Indeed, there may be equilibria that are inefficient or just local minimum points (e.g., some Cournot oligopolies [116]). Let $\mathcal{P}_{\min}$ denote the set of global minima of P on $\mathcal{S}$.

The relationship between the NEs of $\tilde{G}$ (and so of G) and the global minimum of the potential function P on $\mathcal{S}$ is provided by the following theorem.

Theorem 3.10. *Let $G = (\mathcal{N}, \mathcal{S}, (u_i)_{i\in\mathcal{N}})$ be a potential game with potential function P.*

1. *If $x \in \mathcal{P}_{\min}$, then x is a Nash equilibrium of G.*
2. *Assume that $\mathcal{S}$ is a convex set with a Cartesian structure and P is a continuously differentiable and convex function on $\mathcal{S}$. If x is a NE of G, then $x \in \mathcal{P}_{\min}$. If P is strictly convex, the NE is unique.*

These two results imply that we can study, under certain conditions, the properties of NEs using a single function that does not depend on the particular player. Hence, the study of potential games can be carried out using the game $\tilde{G} = (\mathcal{N}, \mathcal{S}, (P)_{i\in\mathcal{N}})$ and the classical game theory framework (Theorem 3.9) or via the standard optimization problem where the objective function is just the potential function (Theorem 3.10):

$$\begin{aligned} &\min_{s} \ P(s) \\ &\text{s.t.} \ \ s \in \mathcal{S} := \mathcal{S}_1 \times \ldots \times \mathcal{S}_N. \end{aligned} \tag{3.33}$$

Note that Theorem 3.10 does not assert the existence of a solution. Nevertheless, the existence of a minimum for the potential function P on the set $\mathcal{S}$ leads directly to the existence of a Nash equilibrium for the game G.

The following theorem describes the conditions for existence and uniqueness of NE in ordinal potential games, based on the characteristics of their strategic spaces and potential functions.

Theorem 3.11 ([117]). *The following statements are true*

- *Every finite (ordinal) potential game admits at least one NE.*
- *Every infinite (ordinal) potential game, whose strategy space $\mathcal{S}$ is compact and potential function P is continuous, admits at least one NE. Moreover, if P is strictly convex, the NE is unique.*

The above results apply only to NEPs. Definitions 3.26-3.28 of potential functions can be extended to GNEPs [118]. In this way, we can establish the following theorem.

Theorem 3.12. *Let $\mathcal{G}$ be a GNEP and a potential game with potential function P (assumed $\mathcal{P}_{\min}$ non-empty). If $x \in \mathcal{P}_{\min}$, then x is a generalized Nash equilibrium of $\mathcal{G}$.*

Remark 3.2. The second point of Theorem 3.10 cannot be applied to potential GNEPs. For clarity, we assume that GNEP is jointly convex. A GNE of the jointly convex GNEP does not necessarily minimize the potential function over the set (3.22). In fact, global constraints shattered the Cartesian structure in the set (3.22) [119, 120].

3.4. Distributed algorithms

At first glance, all we have are theoretical existence results and no explicit method for calculating Nash equilibria of a NEP. In general, the effective determination of these NEs requires appropriate algorithmic methods. A first approach is to use centralized algorithms, where a central operator has complete information on players' objective functions, constraints and strategies.

These algorithms optimize the system as a whole and are well developed in the literature [99, 94], but they can be inappropriate in certain real-life applications.

In the context of energy communities, members are especially concerned about the confidentiality and protection of their data and may be reluctant to share sensitive information. Therefore, the adoption of distributed methods seems more appropriate in this environment. This type of algorithm is better suited to progressive decentralization and the emergence of computing capacities directly on members. It also addresses data confidentiality issues. Members are equipped with smart meters, which not only measure their consumption and production, but also provide communication and processing resources. So, the implementation of distributed algorithms becomes particularly relevant in this setting. This choice eliminates the need for a central authority and thus reinforces the autonomy of end-users within the community.

In this section, we describe two distributed algorithms to solve the game formulation of the energy exchange scheduling problem in energy communities: the best-response and the proximal decomposition algorithms.

Since a NE can be seen as a fixed-point of the best-response mapping for each player $i \in \mathcal{N}$, a natural algorithm is to iterate best-responses and update the strategies of each player simultaneously (Jacobi scheme) or sequentially (Gauss-Seidel scheme), given the strategies of the others. The implementation of the distributed best-response algorithm is described in Algorithm 1.

Algorithm 1 Best-Response Dynamic Algorithm

Choose any feasible point $x^0 \in \Omega$
$k \leftarrow 0$
while a suitable termination criterion is not satisfied **do**
  for $i \in \mathcal{N}$ **do**
    Sequential update:
      $x_i^{k+1} := x^* \in \arg\min_{x_i \in \Omega_i} b_i(x_1^{k+1}, ..., x_{i-1}^{k+1}, x_i, x_{i+1}^k, ..., x_N^k)$
    Simultaneous update:
      $x_i^{k+1} := x^* \in \arg\min_{x_i \in \Omega_i} b_i(x_i, x_{-i}^k)$
  end for
  $k \leftarrow k + 1$
end while

The only computationally demanding step is the computation of the best response $\mathcal{B}_i(x_{-i}^k)$ on line 4. We can used some standard stopping criteria for this algorithm: a maximum number of iterations, an objective on the difference between iterates $\|x^{k+1} - x^k\| \leqslant \varepsilon$, the satisfaction of the KKT conditions up to an error tolerance, etc. However, the fixed-point based analysis can be limited, as it may be difficult to have a closed form calculation of the best response mapping. These issues can be overcome by reducing the NEP to a variational inequality problem.

If the NEP satisfies hypotheses of Proposition 3.1, then we can solve the game by focusing on the associated VI problem, for which several well-established solution methods and convergence of algorithms results are available in the literature. We refer the interested reader to [99]. The convergence properties of the algorithm are provided in the following result [121, 122]. Algorithm 1 globally converges to the solution of the VI problem and so on a NE of the game, under the condition that the VI problem is strongly monotone (Definition 3.23).

Theorem 3.13. *Let $G = (\mathcal{N}, \Omega, b)$ be a NEP such as Proposition 3.1 holds and let $F = (\nabla_{x_i} b_i)_{i=1}^N$. If F is strongly monotone, then any sequence $\{x^k\}_{k=0}^{\infty}$ generated by Algorithm 1 converges to the unique NE of G.*

As a reminder, if a VI problem is strongly monotone, then there is a unique solution, guaranteed by Theorem 3.6. Then, under this setting, the Theorem 3.13 ensures that Algorithm 1 converges to the unique Nash equilibrium.

Example 3.12. For the Cournot competition in Example 3.4, we have shown that there is an equivalent strongly monotone VI problem with a unique solution. The Algorithm 1 is well defined and should converge to the unique NE, according to Theorem 3.13. We study the case where $\alpha = 10, \beta = 5, \gamma_1 = 1$ and $\gamma_2 = 2$, with this value the algorithm should output:

$$q^* = \Big(\frac{\alpha - 2\gamma_1 + \gamma_2}{3\beta}, \frac{\alpha - 2\gamma_2 + \gamma_1}{3\beta}\Big) = \Big(\frac{2}{3}, \frac{7}{15}\Big) \approx (0.668, 0.467).$$

We implemented the problem using the Julia programming language with the Gurobi solver. We take the point $x^0 = (0, 0)$ as initial value and the stopping criterion is set such that the difference between successive iterations had to be less than or equal to 10^{-3}. The algorithm converged to $(0.668, 0.467)$ after 12 iterations and the computation time remained under 1s.

The assumption of strong monotonicity is rather restrictive in practical situations. On the other hand, the monotonicity property is more commonly encountered in real-world applications. Since monotone NEPs can admit multiple equilibria, Algorithm 1 can fail to converge. A wide range of algorithms for solving monotone VI problems are available in the literature, see [99, Vol.II].

However, most of these solution methods are based on centralized approaches. In [122, 123], the authors developed distributed best-response algorithms to solve monotone NEPs with possible multiple solutions. They rely on a regularization technique called proximal algorithms, see [99, Ch 12]. Instead of a single NEP, the approach leads to solving a sequence of strongly convex sub-problems with a particular structure, which are guaranteed to converge under some technical conditions.

Before presenting the formal description of the algorithm, we first highlight a few key observations that provide intuition for the construction of the sequence of strongly monotone NEPs. Let $G = (\mathcal{N}, \Omega, b)$ be a monotone NEP, we consider an additional regularization term in the objective function. The game is defined as $G_{\tau,y} := (\mathcal{N}, \Omega, (b_i + (\tau/2)\|I - y_i\|^2)_{i=1}^N)$, where I is the identity map, τ is a positive parameter and $y = (y_i)_{i=1}^N$ is the center of the regularization with each $y_i \in \mathbb{R}^{n_i}$. The NEP $G_{\tau,y}$ is a game where each player $i \in \mathcal{N}$ solves the following convex optimization problem, given $x_{-i} \in \Omega_{-i}$:

$$\begin{aligned} \min_{x_i} \; & b_i(x_i, x_{-i}) + \frac{\tau}{2}||x_i - y_i||^2 \\ \text{s.t. } \; & \Theta_i \in \Omega_i. \end{aligned} \tag{3.34}$$

The connection between the original and regularized problem is established by the following result.

Proposition 3.4 ([122])**.** *Let $G = (\mathcal{N}, \Omega, b)$ be a monotone NEP. For any given positive τ, $x^* \in \Omega$ is a NE of G if and only if x^* is a NE of G_{τ,x^*}.*

Algorithm 2 describes the *proximal decomposition algorithm* (PDA).

Algorithm 2 Proximal Decomposition Algorithm (PDA)

Choose any starting point $x^0 \in \Omega$. Given $\{\rho^k\}_{k=0}^{\infty}$ and $\tau > 0$.
$k \leftarrow 0$
while a suitable termination criterion is not satisfied **do**
  for $i \in \mathcal{N}$ **do**
    $x_i^{k+1} := x^* \in \arg\min\{b_i(x_i, x_{-i}^k) + \frac{\tau}{2}||x_i - x_i^k||^2, x_i \in \Omega_i\}$
  end for
  if a NE is reached **then**
    each player $i \in \mathcal{N}$ sets $x_i^{k+1} \leftarrow (1 - \rho^k).x_i^k + \rho^k.x_i^{k+1}$
  end if
  $k \leftarrow k + 1$
end while

If τ is large enough, each $G_{\tau,y^{(k)}}$ is a strongly monotone NEP with a unique NE which can be computed by the distributed best-response algorithm 1 whose convergence is guaranteed by Theorem 3.13. The convergence properties of Algorithm 2 are established in the following result.

Theorem 3.14 ([122]). *$G = (\mathcal{N}, \Omega, b)$ be a NEP such as Proposition 3.1 holds and $F = (\nabla_{x_i} b_i)_{i=1}^{N}$ is monotone. Suppose that τ is large enough so that $G_{\tau,y}$ is strongly monotone and choose $\{\rho^k\} \subset [R_m, R_M]$ with $0 < R_m \leqslant R_M < 2$. Then Algorithm 2 is well defined, and the sequence $\{x^k\}_{k=0}^{\infty}$ generated by the algorithm converges to a solution G.*

The two algorithms presented in this section serve as the main computing tools for solving the energy exchange scheduling discussed in Chapter 4.

CHAPTER 4.

Valuing the Electricity Produced Locally in Renewable Energy Communities through Noncooperative Resources Scheduling Games

The design of the RECs studied in our work, implements a collaborative demand-side management scheme inside a community that aims to optimize the use of resources and energy exchanges by unlocking some flexibility, in order to achieve the best objective. This chapter extensively studies the problem of short-term (e.g., day-ahead) dispatch of energy assets within such communities while modeling DSM possibilities. We discuss that a centralized optimization formulation may not be sufficient depending on the local market design, and that game theory is rigorously required: community members share indeed common resources (such as the public power grid, local electricity surpluses) so that members' strategies (e.g., schedule of appliances) may influence the other members' objectives (e.g., individual electricity bills) and feasible strategies. Note that the models developed in this chapter are designed to provide recommendations to the community members.

Further, aware of the increasing willingness of end users to find new alternatives for sourcing their electricity regarding the recent price volatility on electricity markets, we investigate the influence of the retail electricity price on the operation of a REC of domestic users connected to the public LV network.

The content of this chapter is based on the following publications:

- [24] L. Sadoine, M. Hupez, Z. De Grève and T. Brihaye, "Towards Decentralized Models for Day-Ahead Scheduling of Energy Resources in Renewable Energy Communities," in *Operations Research Proceedings 2022*, Springer International Publishing, pp. 321-329, 2023.

- [25] L. Sadoine, Z. De Grève and T. Brihaye, "Impact of retail electricity prices and grid tariff structure on the operation of resources scheduling in Renewable Energy Communities," in *2023 IEEE PES Innovative Smart Grid Technologies Europe (ISGT EUROPE)*, 2023.
- [26] L. Sadoine, Z. De Grève and T. Brihaye, "Valuing the Electricity Produced Locally in Renewable Energy Communities through Noncooperative Resources Scheduling Games," under revision in *Applied Energy*.

4.1. Introduction

Mathematical game theory has been considerably investigated in the Smart Grids literature [10], and more particularly in the framework of DSM modeling at a community or microgrid scale. Noncooperative games provide a convenient framework to model interactions between selfish users sharing a common network. Mohsenian-Rad et al. [9] formulated, for instance, day-ahead energy consumption scheduling games in which each consumer optimizes its own cost by acting on demand scheduling, using a daily billing approach. Atzeni et al. [11, 124] proposed, on the other hand, a DSM scheme consisting of day-ahead optimization, considering both distributed electricity generation and distributed storage as decision variables rather than shifting energy consumption. The problem has been expressed as noncooperative and cooperative games successively, with a continuous billing strategy [11] and has been extended in [124] by embedding global constraints on aggregate bid energy load, thereby creating dependencies between players feasible strategies, leading to a generalized Nash Equilibrium Problem (GNEP). Mishra et al. [125] jointly considered flexible appliances, storage and local dispatchable generation in the DSM process. They solved the game and measured the impact of DSM on system performance parameters. References above do not model, however, in a community framework that is compliant with EU retail tariff regulation (which separates commodity costs from grid costs), and in which economic flows may also be optimized.

Le Cadre et al. [66] analyze, for instance, peer-to-peer energy exchange inside communities with centralized and distributed market designs. Electricity flows are optimized by maximizing the community's social welfare, and the authors formalize a generalized game considering selfish users who optimize their own operation (local demand and flexibility activation). References [63, 126] present a local competitive market for community microgrids, which aims at maximizing the community's social welfare, by formulating the problem as a bilevel model. Hupez et al. [76] proposed a DSM scheme for communities established on typical European LV grids, and focused on the sharing of costs within the community using game-theoretical billings. Hupez et al. [72] considered a collaborative community where surpluses of local renewable generation and excess storage space are made available freely among community members. The latter hypothesis is, however, difficultly justifiable in real communities, in which members who have invested in generation assets may wish to value internally (in the community) and externally (through classical markets) their excess production. Moreover, their network costs do not include capacity-based tariffs.

The allocation of total costs among the members' individual bills is also an important question, which has already been investigated in the literature whether in terms of fairness, user incentives according to their profile or efficiency, see e.g., [9, 76, 72, 24, 127]. However, papers [9, 127] do not fall within the energy communities context, where energy can be exchanged between members. Furthermore, these studies examine the global bill efficiency obtained from the decentralized approach against that obtained with the centralized approach.

In this work, we develop two internal market designs for RECs which dictate the exchanges inside the RECs with flexible appliances, local renewable generation and energy storage systems. The first design **(D1)** implements a collaborative demand-side management scheme inside a community where members' objectives are coupled through grid tariffs, whereas the second design **(D2)** allows the valuation of individual excess generation within the community. The contributions of this chapter are:

1. We extend the formalism of [76, 72] by valuing the electricity exchanged internally and sold on the retail market at non-zero prices, and we augment the grid cost structure by considering peak tariffs. Two grid cost structures are tested, one academic **(T1)** and a realistic one **(T2)** which reflects the Belgian regulations in terms of grid tariffs. We formulate the mathematical problem in a centralized fashion (i.e. optimization-based), and distribute the REC total costs among community members ex-post. We also develop decentralized models based on noncooperative game theory which endogenize cost distribution.

2. We carry out a theoretical and empirical study concerning the existence of equilibria with the decentralized models. We also study the efficiency of the obtained equilibria, i.e., we compare theoretically and empirically the total REC costs obtained at the equilibrium with the social optimum obtained with the centralized formulation. We first show that there always exists an equilibrium that is a social optimum. We also show that the computed equilibrium induces a total bill equal to or slightly different

from the centralized solution, meaning that the faster optimization-based model can be preferred for macroscopic, system-level studies in which communities may be considered as single economic entities.

3. We study and compare the member individual outcomes for the centralized and decentralized formulations, for each cost distribution. We show empirically that replacing decentralization with ex-post allocation from the centralized model essentially keeps the same individual invoices for daily billing methods. The same conclusion applies to continuous pricing if the network costs are linear.

4. We perform a sensitivity analysis of retail electricity prices and measure the impact on the total REC costs with design **D2**. We demonstrate the existence of a threshold in the import retail price, depending on the difference between the import/export community prices and the import/export retail prices, for which the economic gains of operating as a REC increase significantly, for both grid tariffs **T1** and **T2**.

5. We also study the impact on members' individual bills and interest in joining/leaving the community. We show that, according to our hypotheses (rational behavior of members), the realistic grid tariff design **T2** is at least neutral or beneficial in terms of individual costs for each user type, provided certain cost allocation policies in place.

This chapter is organized as follows. Section 4.2 describes the community framework with two , the prosumer load model and the adopted cost structures. Section 4.3 presents the day-ahead energy resources scheduling problem for the design D1 and D2, in both centralized and decentralized cases. We analyze in Section 4.4 the socially optimal solutions and (generalized) Nash equilibria, and propose distributed algorithms for solving the games. The case study data are presented in Section 4.5 and Section 4.6 display outcomes on the proposed use-case. In Section 4.7, we conduct the sensitivity analysis on a REC with design D2 and evaluate outcome differences between centralized and decentralized problems for T1 pricing. On the other hand, numerical results are compared to the case when users act individually without community under T2 pricing. Conclusions constitute the last section.

4.2. Community framework

An overview of the models is presented in Fig.4.1. We assume collaborative communities built on a demand-side management (DSM) scheme and composed

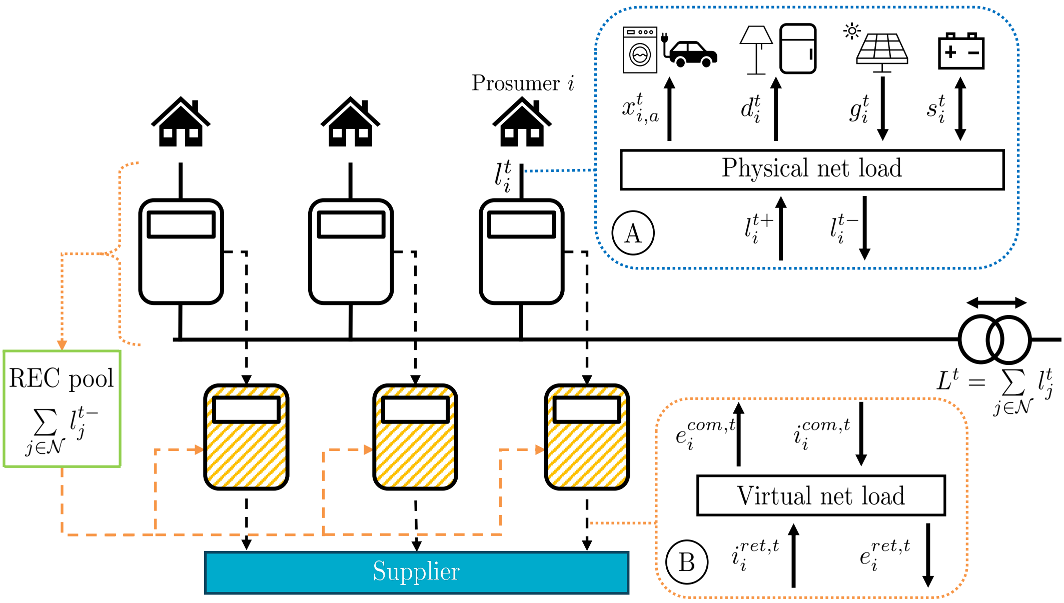


Figure 4.1.: Renewable energy community with (A) physical (design D1 and D2) and (B) virtual decision variables for the pooling of excess generation (design D2).

of consumers and prosumers connected to the same LV public distribution grid, where members may virtually pool their excess production (D2) or not (D1). Each member is equipped with a bidirectional metering device, or smart meter.

4.2.1. Prosumer profile

Let $\mathcal{N} = \{1, \ldots, N\}$ be the set of community members, and $\mathcal{T} = \{1, \ldots, T\}$ the set of time steps of duration Δt within a day. The consumption profile of member $i \in \mathcal{N}$ divides into flexible or non-flexible (base) loads.
The flexible consumption is the load for which end-users consent flexibility in their operation (e.g., electric vehicles, washing machines, etc.). Let A_i be the set of flexible appliances of member i. For each device $a \in A_i$, we define the energy scheduling vector $x_{i,a} = (x_{i,a}^1, \ldots, x_{i,a}^T)$. The non-flexible loads (e.g., fridge) of user i are modeled by $d_i = (d_i^1, \ldots, d_i^T)$ such as $d_i^t \geqslant 0,\ \forall t \in \mathcal{T}$. A user i might also be equipped with non-dispatchable energy generation (e.g., photovoltaic panels), represented by $g_i = (g_i^1, \ldots, g_i^T)$ with $g_i^t \geqslant 0,\ \forall t \in \mathcal{T}$. Note that, we assume a perfect forecast of the non-flexible load and local production.
Each agent may have a personal energy storage system, such as a battery. The power scheduling is given by the storage vector $s_i = (s_i^1, \ldots, s_i^T)$, with $s_i^t > 0$

in charging mode and $s_i^t < 0$ discharging mode.

The physical net load of prosumer i at time $t \in \mathcal{T}$ that is metered by the DSO, is defined as

$$l_i^t = \sum_{a \in A_i} x_{i,a}^t + d_i^t + s_i^t.\Delta t - g_i^t. \tag{4.1}$$

Net load is negative if the prosumer's production exceeds her consumption (export situation), and positive if her local generation does not cover her own consumption (import situation). We define $l_i^{t+} = \max(0, l_i^t)$ and $l_i^{t-} = \max(0, -l_i^t)$, respectively the positive and negative net load, such as $l_i^t = l_i^{t+} - l_i^{t-}$.

The daily peak power consumption of member i reads

$$\max_{t \in \mathcal{T}} \left(\frac{l_i^{t+}}{\Delta t} \right).$$

We avoid the max operator in the objective function by introducing auxiliary variables $\overline{p}_i$, and reformulate as follows:

$$\frac{l_i^{t+}}{\Delta t} \leqslant \overline{p}_i, \quad \forall t \in \mathcal{T}. \tag{4.2}$$

Flexible appliances are subject to constraints. The temporal flexibility consented to device a by individual i is defined by a daily (parameter) binary vector $\delta_{i,a} = (\delta_{i,a}^1, \ldots, \delta_{i,a}^T)$. A value of 1 indicates that member i agrees to schedule a over time slot $t \in \mathcal{T}$, otherwise it will be set to 0. The predetermined total amount of energy that appliance a must consume for the day is denoted $E_{i,a}$. Without loss of generality, we consider flexible devices with fully modular consumption cycles, i.e., each of them is limited only by maximum power $M_{i,a}$, which reads

$$\delta_{i,a}.x_{i,a}^\top = E_{i,a} \tag{4.3}$$

$$0 \leqslant x_{i,a}^t \leqslant M_{i,a}.\delta_{i,a}^t.\Delta t, \quad \forall t \in \mathcal{T}. \tag{4.4}$$

We adopt a simplified storage model neglecting all losses. The battery is subject to maximum charge M_i^{ch} and discharge M_i^{dis} power levels, which yields

$$-M_i^{\text{dis}} \leqslant s_i^t \leqslant M_i^{\text{ch}} \quad \forall t \in \mathcal{T}. \tag{4.5}$$

The initial state of charge e_i^0 is expressed as a percentage of battery capacity. We impose that the final state of charge is equal to this value. The storage capacity is noted E_i^{st}. The battery state of charge constraints are written:

$$0 \leqslant e_i^0 + \sum_{h=1}^{t} s_i^h.\Delta t \leqslant E_i^{st}, \quad \forall t \in \mathcal{T} \tag{4.6}$$

$$e_i^0 + \sum_{t \in \mathcal{T}} s_i^t.\Delta t = e_i^0. \tag{4.7}$$

We consider maximum injection and withdrawal connection power for each member

$$l_i^{t+} \leqslant l_i^{max}, \quad \forall t \in \mathcal{T} \tag{4.8}$$

$$l_i^{t-} \leqslant g_i^t, \quad \forall t \in \mathcal{T} \tag{4.9}$$

$$\overline{p}_i \leqslant \frac{l_i^{max}}{\Delta t}, \tag{4.10}$$

where $l_i^{max} > 0$ is the upper bound of the member's capacities.

In the case of design D2, which allows for the virtual mutualization of excess resources among community members (see Fig. 4.1), we further define virtual power flows[1] that deviate from the physical flows. A prosumer $i \in \mathcal{N}$ with a production surplus may sell a quantity $e_i^{com,t}$ at time step $t \in \mathcal{T}$ to the community

$$0 \leqslant e_i^{com,t} \leqslant l_i^{t-}, \quad \forall t \in \mathcal{T}. \tag{4.11}$$

A member i in an energy deficiency situation may also purchase energy $i_i^{com,t}$ from the community

$$0 \leqslant i_i^{com,t} \leqslant l_i^{t+}, \quad \forall t \in \mathcal{T}. \tag{4.12}$$

Moreover, the total excess production allocated to the community must equal the total quantity imported by members at each time step:

$$\sum_{i \in \mathcal{N}} i_i^{com,t} = \sum_{i \in \mathcal{N}} e_i^{com,t}, \quad \forall t \in \mathcal{T}. \tag{4.13}$$

Finally, the volumes imported $i_i^{ret,t}$ from and exported $e_i^{ret,t}$ to the retail market by member i are obtained by:

$$i_i^{ret,t} = l_i^{t+} - i^{com,t}, \quad \forall t \in \mathcal{T} \tag{4.14}$$

$$e_i^{ret,t} = l_i^{t-} - e_i^{com,t}, \quad \forall t \in \mathcal{T}. \tag{4.15}$$

[1] representing commercial, monetary-based exchanges.

In summary, if the net load is positive, the energy is imported from the REC pool $i_i^{com,t}$ and/or from the supplier $i_i^{ret,t}$. Similarly, if the net load is negative, the energy surplus can be sold to other members $e_i^{com,t}$ and/or to the supplier $e_i^{ret,t}$.

The decision variables set of a user $i \in \mathcal{N}$ is defined as $\Theta_i = \{x_{i,a}, s_i, l_i^+, l_i^-, \overline{p}_i\}$ in D1 model and $\Theta_i = \{x_{i,a}, s_i, l_i^+, l_i^-, i_i^{com}, e_i^{com}, i_i^{ret}, e_i^{ret}, \overline{p}_i\}$ for D2 model, with $\Theta := (\Theta_1, \ldots, \Theta_N)$. Note that the number of decision variables for a member $i \in \mathcal{N}$ can be expressed as follows:

$$n_i := T.(|\mathcal{A}_i| + \kappa) + 1, \tag{4.16}$$

with $\kappa = 3$ for model D1 and $\kappa = 7$ for model D2.

4.2.2. Cost structure

We assume that members aim to minimize their energy bill. The community electricity bill is constituted by different components:

- **Gray energy costs (D1, D2)**: these costs are based on the portion of consumption not covered locally, they are charged by the electricity supplier. We assume a single supplier for the whole community, without loss of generality. For each member $i \in \mathcal{N}$, the costs are formulated as $C_{ret,i}^t = \lambda_{imp}^t . l_i^{t+}$ (D1 model) or $C_{ret,i}^t = \lambda_{imp}^t . i_i^{ret,t}$ (D2 model), with λ_{imp}^t the retailer's import price in €/kWh.

- **Local energy costs (D2 only):** costs of electricity bought from the REC pool, at tariff λ_{iloc}^t €/kWh. For each user $i \in \mathcal{N}$, we have $C_{loc,i}^t = \lambda_{iloc}^t . i_i^{com,t}$.

- **Revenue from exported energy (D1, D2):** income related to the sale of excess local production on the retail market $R_{ret,i}^t = \lambda_{exp}^t . l_i^{t-}$ (D1 model) or $R_{ret,i}^t = \lambda_{exp}^t . e_i^{ret,t}$ (D2 model), with λ_{exp}^t the retailer export price in €/kWh. In D2 model only, revenues for the energy exported on the REC pool are $R_{loc,i}^t = \lambda_{eloc}^t . e_i^{com,t}$, with λ_{eloc}^t the community export price in €/kWh.

- **Grid costs (D1, D2):** costs related to upstream transmission and distribution grid utilization. These include:
  - Volumetric-based costs: we assume two grid pricing structures. As in [76], **Tariff T1** translates the upstream grid usage as a quadratic cost term expressed as $C_{gr}^{\mathrm{T1},t} = \alpha.(L^t)^2$, with $L^t = \sum_{i\in\mathcal{N}} l_i^t$ the aggregate

net load at the MV/LV transformer and α €/kWh^2. **Tariff T2** is in line with the real grid tariffs applied in Flanders (Belgium) [128], to which we add a possible discount for the energy consumed locally as in Brussels (Belgium). This linear cost term is $C_{gr}^{\mathrm{T2},t} = \alpha. \sum_{i\in\mathcal{N}}(i_i^{ret,t} + \gamma.i_i^{com,t})$, with $\gamma \in [0,1]$ discount factor.

– The peak-based costs: are based on the daily [2] peak power consumption of each member as $C_{p,i} = \beta.\overline{p}_i$, with β €/kW unit penalty cost.

The total REC costs for D1 and D2 are respectively:

$$f^{\mathrm{D1}}(\Theta) = \sum_{t\in\mathcal{T}}\Big[\sum_{i\in\mathcal{N}}(C_{ret,i}^t - R_{ret,i}^t) + C_{gr}^t\Big] + \sum_{i\in\mathcal{N}} C_{p,i} \tag{4.17}$$

$$f^{\mathrm{D2}}(\Theta) = \sum_{t\in\mathcal{T}}\Big[\sum_{i\in\mathcal{N}}(C_{ret,i}^t + C_{loc,i}^t - R_{ret,i}^t - R_{loc,i}^t) + C_{gr}^t\Big] + \sum_{i\in\mathcal{N}} C_{p,i} \tag{4.18}$$

We assume $\lambda_{exp}^t < \lambda_{imp}^t$ and $\lambda_{eloc}^t < \lambda_{iloc}^t$ for all $t \in \mathcal{T}$.

4.3. Day-ahead energy resources scheduling problem

We formulate the day-ahead energy exchange scheduling problem, in which members optimize their available flexibility (D1 and D2) and the virtual energy exchanges (D2) so as to minimize the total energy costs. Section 4.3.1 presents the centralized (i.e., optimization-based) formulation, whereas Section 4.3.2 describes the decentralized (i.e., game theoretical) formulation.

4.3.1. Centralized optimization models

This section describes the optimization-based formulation for both designs D1 and D2. We refer the reader to Section 3.1 of Chapter 3 concerning optimization theory.

Design D1 consists in a collaborative demand-side management scheme between community members, coupled via the upstream grid cost component. We assume a central operator (e.g., a community manager) is solving the

[2]In [128], the mean month power peak is charged. Since it would make no sense to plan energy resources hourly over a month, we assumed that minimizing the daily peak is equivalent to minimize the mean monthly peak. We used a daily peak tariff component, and adjusted the unit price accordingly.

optimization model in order to minimize the total REC electricity costs

$$P_1 := \begin{cases} \min\limits_{\Theta} \; f^{\mathrm{D1}}(\Theta) \text{ as in (4.17)} \\ \text{s.t. } \; \Theta \in \Omega^1 \end{cases} \tag{4.19}$$

with $\Omega^1 := \left\{(x_i, s_i, l_i^+, l_i^-, \overline{p}_i)_{i=1}^N \in \mathbb{R}^n \mid (4.1) - (4.10)\right\}$ the feasible set and $n := \sum_{i \in \mathcal{N}} n_i$.

Design D2 allows prosumers to sell their excess production to the community pool, whereas consumers may purchase electricity on that pool. We have

$$P_2 := \begin{cases} \min\limits_{\Theta} \; f^{\mathrm{D2}}(\Theta) \text{ as in (4.18)} \\ \text{s.t. } \; \Theta \in \Omega^2 \end{cases} \tag{4.20}$$

where $\Omega^2 := \left\{(x_i, s_i, l_i^+, l_i^-, i_i^{com}, e_i^{com}, i_i^{ret}, e_i^{ret}, \overline{p}_i)_{i=1}^N \in \mathbb{R}^n \mid (4.1) - (4.15)\right\}$ is the feasible set.

The profiles minimizing the total bill are named *socially optimal solutions*. We define $X_{\text{opt}}(P_1)$ and $X_{\text{opt}}(P_2)$ the socially optimal solution sets of the optimization problems P_1 and P_2 respectively.

4.3.2. Noncooperative games

Designs D1 and D2 may give rise to strategic interactions between community members, who compete for common resources (i.e., the electrical network and the local production surplus), which are not captured by (4.19)-(4.20). Indeed, each member has control over her load profile that impacts her own aims, which may conflict with those of the other users. Additionally, the allocation of costs to each member is not addressed in the centralized models, which minimize the total REC electricity bill only (cost allocation must be performed ex-post in that case). We formulate, consequently, the day-ahead energy resources scheduling problem as a noncooperative game for D1 and D2, where the cost distribution is endogenized in the members' objective functions. We propose four mechanisms for allocating REC costs among individuals and computing the individual bills b_i of each member $i \in \mathcal{N}$, inspired by [76].

The first three cost distribution methods share the total REC daily costs $f(\Theta)$ proportionally among members according to distribution keys K_i. Then, these keys of distribution have constant values over the horizon time and the sum of these keys is 1. The last billing method allocates the total REC costs on an hourly basis. Note that other allocation mechanisms, based on energy sharing rather than cost sharing, can be found in the literature (see e.g., [29, 60]).

1. **Egalitarian billing [EB]**. The first static key is simply based on equal sharing between members of the community.

$$b_i^{\mathrm{EB}}(\Theta) = K_i f(\Theta) = \frac{1}{N} f(\Theta), \quad \forall i \in \mathcal{N}. \tag{4.21}$$

2. **Net load proportional billing [NET].** The distribution key for member $i \in \mathcal{N}$ is given by the ratio between the absolute value of her net load and the community net load. More precisely, we consider the minimal value of the net load, i.e. $l_i^* \in \mathrm{argmin}(\sum_{t\in\mathcal{T}} |l_i^t|)$, in order to make the key independent of the solution set [76].

$$b_i^{\mathrm{NET}}(\Theta) = K_i f(\Theta) = \frac{\sum_{t\in\mathcal{T}} l_i^{t*}}{\sum_{j\in\mathcal{N}} \sum_{t\in\mathcal{T}} l_j^{t*}} f(\Theta), \quad \forall i \in \mathcal{N}. \tag{4.22}$$

3. **Marginal cost billing [VCG].** The distribution key of each member is calculated using the normalized Vickrey-Clarke-Groves (VCG) mechanism [127, 76]. A new approach addresses the absolute contribution of the user:

$$b_i^{\mathrm{VCG}}(\Theta) = K_i f(\Theta) = \frac{|C_{\mathcal{N}}^* - C_{\mathcal{N}\setminus\{i\}}^*|}{\sum_{j\in\mathcal{N}} |C_{\mathcal{N}}^* - C_{\mathcal{N}\setminus\{j\}}^*|} f(\Theta), \quad \forall i \in \mathcal{N} \tag{4.23}$$

   where $C_{\mathcal{N}}^*$ are the minimum community costs for a REC composed by all members of set $\mathcal{N}$, and are obtained by solving (4.19) or (4.20). This requires the solution of N additional optimization problems to obtain each $C_{\mathcal{N}\setminus\{i\}}^*$.

4. **Continuous billing [CB].** The total cost is distributed among community members at each time slot $t \in \mathcal{T}$. We have for designs D1 and D2 respectively, for each user $i \in \mathcal{N}$

$$b_i^{\mathrm{CB1}}(\Theta) = \sum_{t\in\mathcal{T}} (C_{ret,i}^t - R_{ret,i}^t + C_{gr,i}^t) + C_{p,i} \tag{4.24a}$$

$$b_i^{\mathrm{CB2}}(\Theta) = \sum_{t\in\mathcal{T}} (C_{ret,i}^t + C_{loc,i}^t - R_{ret,i}^t - R_{loc,i}^t + C_{gr,i}^t) + C_{p,i} \tag{4.24b}$$

   where $C_{gr,i}^t = l_i^t.\alpha L^t$, with $L^t = \sum_{j\in\mathcal{N}} l_j^t$, for tariff T1 and $C_{gr,i}^t = \alpha.(i_i^{ret,t} + \gamma.i_i^{com,t})$ for tariff T2.

Design D1 as a NEP

We rewrite the problem (4.19) as a Nash equilibrium problem (NEP) described in Section 3.2.2 of Chapter 3. Each member $i \in \mathcal{N}$ is a selfish player choosing her strategy $\Theta_i \in \mathbb{R}^{n_i}$ in order to minimize her own daily cost function $b_i : \mathbb{R}^n \to \mathbb{R}$, which depends itself on other players' strategies $\Theta_{-i} := (\Theta_1, \ldots, \Theta_{i-1}, \Theta_{i+1}, \ldots, \Theta_N)$. Mathematically, each member solves the following optimization problem, given Θ_{-i} the other players' strategies

$$G := \begin{cases} \min\limits_{\Theta_i} & b_i(\Theta_i, \Theta_{-i}) \qquad \forall i \in \mathcal{N} \\ \text{s.t.} & \Theta_i \in \Omega_i \end{cases} \tag{4.25}$$

where $\Omega_i \subseteq \mathbb{R}^{n_i}$ is the strategy set constituted by the player i's individual constraints, which are independent of the other members' strategies in D1. The n-dimensional joint strategy set is expressed as $\Omega := \prod_{i \in \mathcal{N}} \Omega_i$, with $n := \sum_{i \in \mathcal{N}} n_i$. The game G is described as the triplet $G = (\mathcal{N}, \Omega, (b_i)_{i=1}^N)$.

A strategy profile $\Theta^* \in \Omega$ is called a Nash equilibrium (NE) of the game G (4.25) if $\forall i \in \mathcal{N}$:

$$b_i(\Theta_i^*, \Theta_{-i}^*) \leqslant b_i(\Theta_i, \Theta_{-i}^*), \quad \forall \Theta_i \in \Omega_i. \tag{4.26}$$

The set of NEs of game G is denoted NE(G). A NE is a feasible strategy profile such that no single player can benefit by unilaterally deviating from her strategy.

The REC's total cost is the sum of the members' individual energy bills, which, by Definition, 3.15 corresponds to the social cost. A strategy profile is a social optimum if it optimizes social costs. Note that a NE does not necessarily lead to a social optimum, and a social optimum may not be a NE. A game G is said to be equivalent to a minimization problem of a function $F : \mathbb{R}^n \to \mathbb{R}$ if, for all $\Theta^* \in \Omega$, Θ^* is a NE if and only if Θ^* is a global minimum of F.

Design D2 as a GNEP

Design D2 allows excess production to be shared among community members. This translates into additional constraints ensuring the balance of internal virtual exchanges (4.13). These constraints couple the strategy set of each player to her rivals' decisions. We reformulate consequently problem (4.20) as a generalized Nash equilibrium problem (GNEP) described in Section 3.2.4 of Chapter 3. In a GNEP, both the objective functions and the strategy sets depend on the rivals' strategies, contrary to NEPs where interactions occur on

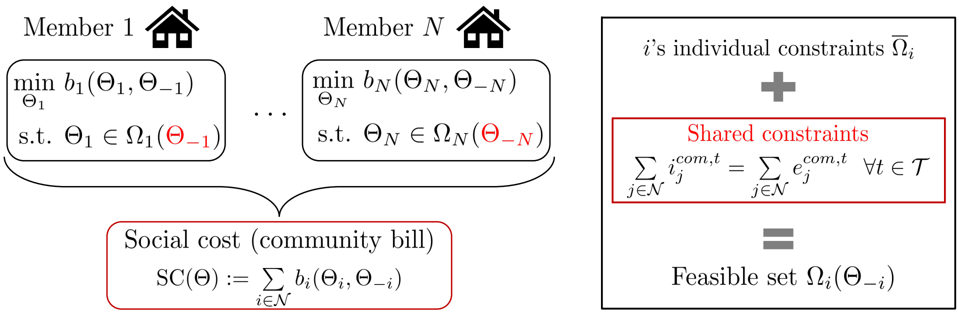


Figure 4.2.: Design D2 as a generalized Nash equilibrium problem.

the objective functions only. We have

$$\mathcal{G} := \begin{cases} \min\limits_{\Theta_i} & b_i(\Theta_i, \Theta_{-i}) \qquad \forall i \in \mathcal{N} \\ \text{s.t.} & \Theta_i \in \Omega_i(\Theta_{-i}) \end{cases} \tag{4.27}$$

where the strategies of player $i \in \mathcal{N}$ must belong to her feasible strategy set $\Omega_i(\Theta_{-i}) \subseteq \mathbb{R}^{n_i}$. Similarly, the game is described as the triplet $\mathcal{G} = (\mathcal{N}, (\Omega_i(\Theta_{-i}))_{i\in\mathcal{N}}, (b_i)_{i\in\mathcal{N}})$.

In our case, the members share common linear equality constraints, so that we define the feasible set of agent $i \in \mathcal{N}$ as

$$\Omega_i(\Theta_{-i}) := \left\{\Theta_i \in \overline{\Omega}_i \mid h(\Theta_i, \Theta_{-i}) = 0\right\}, \tag{4.28}$$

where $\overline{\Omega}_i \subseteq \mathbb{R}^{n_i}$ is player i individual constraints set. We write the shared coupling constraints (4.13) as: $h(\Theta) := (\sum_{i\in\mathcal{N}} e_i^{com,t} - i_i^{com,t})_{t\in\mathcal{T}}$. The joint strategy set reads

$$\mathcal{C} = \left\{\Theta \in \mathbb{R}^n \mid \Theta_i \in \overline{\Omega}_i \ \forall i \in \mathcal{N},\ h(\Theta) = 0\right\}. \tag{4.29}$$

A strategy profile Θ^* is called a generalized Nash equilibrium (GNE) of the game $\mathcal{G}$ (4.27) if $\forall i \in \mathcal{N}$:

$$b_i(\Theta_i^*, \Theta_{-i}^*) \leqslant b_i(\Theta_i, \Theta_{-i}^*), \quad \forall \Theta_i \in \Omega_i(\Theta_{-i}^*). \tag{4.30}$$

The set of generalized Nash equilibria of $\mathcal{G}$ reads GNE($\mathcal{G}$). As with NEPs, the community's total bill corresponds to the social cost, and a GNE is not automatically a social optimum, just as a social optimum may not be an equilibrium.

4.4. Analysis and resolution

This section studies the existence of solutions and presents solution algorithms for each model presented in Section 4.3. It also provides a theoretical analysis for assessing the overall and individual efficiency of the various billing mechanisms. We use the Price of Anarchy (Definition 3.16) and the Price of Stability (Definition 3.17) as performance measures to estimate the performance of decentralized models in comparison with the centralized formulations, which, in fact, correspond to the problems of minimizing the social cost. The theoretical results obtained in sections 4.4.2 and 4.4.3 are summarized and supported empirically in Table 4.6 on page 113, Section 4.6.2 of the present chapter.

4.4.1. Optimization problems

We first analyze the classification and properties of centralized optimization problems P_1 (4.19) and P_2 (4.20) for both grid tariff structures.

Theorem 4.1. *The optimization problems P_1 (4.19) and P_2 (4.20), with T1 or T2, have at least a global minimum.*

These models are convex quadratic optimization (tariff T1) or linear optimization (tariff T2) problems, for which the existence of a solution is guaranteed. Multiple solutions may exist, however, since the objective functions f^{D1} and f^{D2} are not strongly convex. This could lead to a fairness issue among members, as some may prefer one solution, while others may prefer another. These problems can be solved in a centralized way by standard algorithms, such as interior-point methods, in polynomial time [94, 129].

Remark 4.1. At the optimum, no simultaneous export to import from the grid can occur for member $i \in \mathcal{N}$ over a time period $t \in \mathcal{T}$. The same holds for the energy imported from and exported to the community. It can be shown via the KKT conditions of the convex optimization problems.

4.4.2. Nash equilibrium problems

We study Nash equilibria in the case of design D1 (section 4.3.2), for tariff T1 and each cost distribution method. Case T2 is discussed in remark 4.2. The properties of our problem allow us to draw connections with other theories presented in Chapter 3. In this way, we can establish the following results.

Theorem 4.2. *Given the NEP (4.25) with tariff T1, for the four cost distribution methods, the NEs set of the game is nonempty and compact.*

Proof. We provide the proof in Appendix A.1. ■

Although Theorem 4.2 guarantees the existence of an equilibrium, there may be multiple solutions, which are discussed in Theorem 4.3.

Theorem 4.3. *Given the NEP G defined in* (4.25),

1. *For [EB,NET,VCG], the game is equivalent to the P_1 problem* (4.19). *Each Nash equilibrium Θ^* of G is a social optimum and leads to the same values of the individual objective functions:*

$$\forall \Theta^*, \Theta' \in \mathrm{NE}(G), \forall i \in \mathcal{N},\ \ b_i(\Theta^*) = b_i(\Theta').$$

2. *For [CB], the game is equivalent to the optimization problem:*

$$\begin{aligned} &\min_{\Theta}\ f^{\mathrm{D1}}(\Theta) - \frac{\alpha}{2}\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{N}} l_i^t . L_{-i}^t \\ &\text{s.t.} \Theta \in \Omega \end{aligned} \tag{4.31}$$

where $L_{-i}^t = \sum_{j\in\mathcal{N}\setminus\{i\}} l_j^t$.

Proof. We provide the proof in Appendix A.1. ■

Theorem 4.3 states that NEP (4.25) can always be formulated as a centralized optimization problem. More particularly, for [EB,NET,VCG] cost distribution methods, the NE set coincides with the optimal solutions of the centralized problem P_1, i.e., $\mathrm{NE}(G) = X_{\mathrm{opt}}(P_1)$. The values of players' billing functions are furthermore constant over the NE set. Roughly speaking, one can say that all the NEs are equivalent in terms of values of the players' cost functions. As a direct consequence of the Theorem 4.3.1, we deduce the Corollary 4.1 which reads that all NEs are efficient.

Corollary 4.1. *Consider the NEP G defined in 4.25, and further assume that the cost allocation is based on [EB,NET,VCG], then*

$$\mathrm{PoA}(G) = \mathrm{PoS}(G) = 1.$$

In the case of [CB], the NEs may not achieve social optimality, as we can observe for our use-case in Section 4.6.2. However, we certainly cannot theoretically assert that this will never happen for any of them.

Nevertheless, a centralized resolution raises important issues related to prosumers' consumption privacy. We focus on the computation of the NEs of game

(4.25), via distributed iterative algorithms discussed in Section 3.4 of Chapter 3. Since the problem may have multiple NEs, the classical best-response algorithm (see Algorithm 1) may fail to converge [122]. Hence, we adopt a proximal decomposition algorithm (PDA) (see Algorithm 2), that have desirable privacy-preserving properties [11, 122]. Indeed, each member solves its local problem and only need the aggregate net load which can be exchanged between community members' smart meters, without needing to be stored or made accessible to any other party. The sequence generated by the PDA converges to a solution of the game. More details regarding the algorithm and its convergence are provided in Appendix A.2.

Remark 4.2. If we apply continuous cost allocation [CB] with T2 pricing, there is no dependency between players in the objective function. We end up with N independent optimization problems, which is not a game. For daily billings [EB, NET, VCG], results remain similar to T1.

As a summary, this theoretical study indicates that for any cost distribution key, we can always write an optimization model, which is equivalent to the NEP, and that the NEP resolution via PDA converges towards a game equilibrium. For the daily billings [EB, NET, VCG], all NEs further respect the community-level efficiency and do not induce any deviations in individual invoices.

4.4.3. Generalized Nash equilibrium problems

The power balance conditions within the community (4.13) imply that the strategy sets depend on the other players' strategies. As a result, GNEPs are more complicated to solve than NEPs. This section examines the equilibria of GNEP (4.27), with both designs T1 and T2.

Problem $\mathcal{G}$ in (4.27) belongs to the *jointly convex* GNEPs subclass, defined in Definition 3.20. For jointly convex GNEPs, one can characterize a subset of GNE, presenting desirable properties for our application, named *variational equilibria* (VEs) (see Definition 3.25, [106]). Those concepts are defined in Chapter 3 and discussed for our framework in Appendix B. Recall that we note VE($\mathcal{G}$) the set of VEs of a jointly convex GNEP $\mathcal{G}$.

One can show that a VE (and thus a GNE) always exists.

Theorem 4.4. *Let us consider GNEP* (4.27) *for both tariffs T1 and T2, as well as all cost distribution methods. The game possesses at least one generalized Nash equilibrium:* $\mathrm{GNE}(\mathcal{G}) \neq \emptyset$.

Proof. We provide the proof in Appendix B.1. ■

Theorem 4.4 guarantees the GNE existence for game $\mathcal{G}$, but it may have multiple solutions. In contrast to the case discussed in Section 4.4.2, it is theoretically not possible to provide a full characterization of the GNEs. However, depending on the pricing system, we may still analyze some of them to determine the most appealing.

Theorem 4.5. *Given the jointly convex GNEP* (4.27)*, for both tariffs T1 and T2:*

1. *For [NET,VCG], the social optimum of P_2 in* (4.20) *are included in the GNEs set, i.e., $X_{\text{opt}}(P_2) \subseteq \text{GNE}(\mathcal{G})$.*
2. *For [EB], the VEs set coincides with the social optimum of P_2 over the set $\mathcal{C}$, and leads to the same value of the objective functions.*

$$\forall \Theta^*, \Theta' \in \text{VE}(G), \forall i \in \mathcal{N},\ b_i(\Theta^*) = b_i(\Theta').$$

For [CB] with the billing function (4.24b)*,*

3. *In the case of T1 quadratic pricing, the VEs set coincides with the optimal solutions of the optimization problem:*

$$\begin{aligned} &\min_{\Theta}\ f^{\text{D2}}(\Theta) - \frac{\alpha}{2}\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{N}} l_i^t.L_{-i}^t \\ &\text{s.t.}\Theta \in \mathcal{C} \end{aligned} \tag{4.32}$$

   where $L_{-i}^t = \sum_{j\in\mathcal{N}\setminus\{i\}} l_j^t$.

4. *In the case of T2 linear pricing, the VEs set coincides with the social optimum of P_2 over the set $\mathcal{C}$, i.e.,*

$$X_{\text{opt}}(P_2) = \text{VE}(\mathcal{G}) \subseteq \text{GNE}(\mathcal{G}).$$

Proof. We provide the proof in Appendix B.2. ■

From Theorem 4.5, we can deduce the following corollary.

Corollary 4.2. *Given the jointly convex GNEP $\mathcal{G}$ defined in 4.27, for both tariffs T1 and T2:*

1. *If we assume [NET,VCG] then the Price of Stability of $\mathcal{G}$ is equal to 1.*
2. *For [EB], we have that the Price of Stability of $\mathcal{G}$ is equal to 1, and the Price of Anarchy restricted to game's variational equilibria is also equal to 1.*

In the case of T2 linear pricing only:

3. *For [CB], we have that the Price of Stability of $\mathcal{G}$ is equal to 1, and the Price of Anarchy restricted to game's variational equilibria is also equal to 1.*

For [EB,NET,VCG], we note that all the optimal solutions to the social problem (4.20) are included in the GNEs set. Therefore, the game with any of these three cost distribution methods has its Price of Stability equal to 1. We can be more specific in the case of [EB], as the optimal solutions to the centralized problem coincide with the VEs of the game. Moreover, all VEs are equivalent. Besides, the game with [CB] and realistic T2 pricing also has a Price of Stability equal to 1 and its VEs coincide with the optimal solutions for the P_2 optimization problem. On the other hand, there may be GNEs which are suboptimal in regard to the socially optimal cost. Note that for [CB] with T1 pricing, nothing can be asserted theoretically about equilibria efficiency, thus GNEs and VEs may not achieve social optimality.

Even though Theorem 4.4 ensures the existence of an equilibrium, we need to be able to calculate it in practice. The optimal selection and monitoring of GNEs is still considered as a scientific challenge and goes beyond the scope of this work [130]. Here, we resort to the proximal decomposition algorithm with shared constraints [122]. This algorithm is guaranteed to converge towards a VE for all cost distribution methods using tariff T2 and [EB,CB] for tariff T1. Thus, we are sure to obtain an efficient equilibrium for [EB] and [CB] under tariff T2. The [NET,VCG] are more sensitive to T1 pricing, however, we empirically observe the convergence to a GNE for our use-case in Section 4.6. We provide more details regarding the convergence of the algorithm in Appendix B.3.

In summary, at least one social optimal GNE exists, except for [CB] in design T1. The PDA with shared constraints converges towards an equilibrium corresponding to the social optimum for [EB] with T1 and T2, and [CB] with design T2. In theory, nothing can be asserted about the solution efficiency of allocation methods [NET,VCG]. However, the empirical study conducted in Section 4.6 reveals that inefficiency is negligible for both keys for our use case.

4.5. Case study

4.5.1. Members profiles and parameters

We study a REC composed of 55 residential members who mutualize excess PV generation (design D2) and connected behind the same MV/LV feeder. For the non-flexible loads, hourly electricity consumption profiles are extracted from the Pecan Street Project dataset [131] and generated for whole days, with $T = 24$. Battery Energy Storage Systems are assigned to community members with a penetration level of 50%. The initial battery state-of-charge is fixed at 50%. Installed PV capacities vary between 0 and 10 kWp. The day ahead energy scheduling models are run for 20 days (10 days with high PV production - 10 days with low PV). We assume that the total daily energy demand remains constant, regardless of the schedule of flexible appliances (no load shedding). The prosumers may own different flexible devices: white goods (dishwashers, washing machines, clothes dryers, etc.), Electric Vehicles (EV) and Heat Pump (HP). For simplicity and without loss of generality, the latter is considered as a fully flexible load. We consider bi-hourly commodity tariffs: $\lambda^t_{imp} = 0.08$ €/kWh, $\lambda^t_{exp} = 0.02$ €/kWh, $\lambda^t_{iloc} = 0.065$ €/kWh and $\lambda^t_{eloc} = 0.032$ €/kWh between 21 p.m. and 4 a.m., and $\lambda^t_{imp} = 0.16$ €/kWh, $\lambda^t_{exp} = 0.04$ €/kWh, $\lambda^t_{iloc} = 0.13$ €/kWh and $\lambda^t_{eloc} = 0.05$ €/kWh elsewhere. We assume constant network tariffs with $\alpha = 0.00109488$ €/kWh^2 for T1 grid tariffs and $\beta = 0.1096737$ €/kW. In T2 pricing, we take $\alpha = 0.027$ €/kWh and a $\gamma = 0.5$ discount on the tariff grid for energy imported from the REC pool.

4.5.2. Benchmark and key performance indicators

We simulate the REC use-case under designs D1 and D2, for both grid tariff structures T1 and T2. We compare outcomes with a benchmark where each user $i \in \mathcal{N}$ individually minimizes her own commodity and peak power costs (situation without community). In each case, the REC total and the individual bills are calculated according to the cost distribution method selected from those presented in Section 4.3.2. In the T1 grid pricing benchmark, total costs are gathered by summing over the individual costs of each user first, and then adding the upstream network volumetric costs. The individual costs are obtained directly when using the game-theoretical models, or computed ex-post with the centralized optimization models.

We further calculate the following technical Key Performance Indicators (KPIs):

- **Self-Consumption Rate (SCR):** ratio between the production consumed locally and the total production

$$\text{SCR} := 1 - \frac{\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{N}} \kappa_i^t}{\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{N}} g_i^t}, \text{ with } \kappa_i^t = \begin{cases} l_i^{t-} & \text{if design D1} \\ e_i^{ret,t} & \text{if design D2.} \end{cases} \tag{4.33}$$

- **Self-Sufficient Rate (SSR):** ratio between the load supplied locally and the total consumption

$$\text{SSR} := 1 - \frac{\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{N}} \kappa_i^t}{\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{N}} l_i^t + g_i^t}, \text{ with } \kappa_i^t = \begin{cases} l_i^{t+} & \text{if design D1} \\ i_i^{ret,t} & \text{if design D2.} \end{cases} \tag{4.34}$$

- **Peak to Average Ratio$^{+(-)}$ (PAR$^{+(-)}$):** ratio between the peak community consumption(+)/injection(-) and the average community consumption/injection

$$\text{PAR}^{+(-)} := \frac{T.\max_{t\in\mathcal{T}}\sum_{i\in\mathcal{N}} l_i^{t+(-)}}{\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{N}} l_i^{t+(-)}}. \tag{4.35}$$

All the convex quadratic and linear problems arising from the models are coded in Julia/JuMP [22] and solved using Gurobi [23]. We report CPU solving times.

4.6. Results and discussion

We first study the centralized, optimization-based formulations for designs D1 (4.19) and D2 (4.20), and compare with the individual benchmark. We then provide a detailed analysis of NEP (4.25) under T1 pricing, and GNEP (4.27) for both grid tariff structures. In each subsection, we start by summarizing our findings and then illustrate and discuss supporting results. We finally expose a synthesis of our theoretical and experimental findings for noncooperative games, along with two observations of important practical significance. Simulations have been performed on a laptop with an Intel(R) Core(TM) i7-10750H processor and 16 GB of RAM.

4.6.1. Centralized formulations

Design D2 which includes the valuation of excess local generation, and to a lesser extent design D1, tends to lower the total REC electricity costs and to improve the technical KPIs.

We compare the optimal solution for the community as a whole for models D1 and D2 as presented in Section 4.3.1, and across different grid tariff structures. Table 4.1-4.2 depicts the mean and standard deviations (between parentheses) of the total REC costs and technical KPIs, over the 10 high and low PV generation days. The total REC cost components are shown in Fig. 4.3. CPU times for solving the model for one day are equal to 1s for design D1 (6,631 decision variables and 21,267 constraints), and 2s for design D2 (11,911 variables and 29,235 constraints).

	REC Bill [€]	PAR+	PAR-	SCR [%]	SSR [%]
Day			High PV		
Benchmark	317.15 (13.24)	2.37 (0.03)	4.44 (0.28)	73.08 (1.4)	47.49 (0.78)
Design D1	221.55 (12.12)	1.19 (0.03)	2.45 (0.07)	73.08 (1.4)	47.49 (0.78)
Design D2	203.02 (13.42)	1.19 (0.03)	2.46 (0.07)	100 (0)	65 (2.3)
Day			Low PV		
Benchmark	819.55 (16.83)	2.22 (0.01)	4.8 (10.12)	99.99 (0.03)	1.49 (0.4)
Design D1	664.65 (12.42)	1.21 (0.01)	4.8 (10.12)	99.99 (0.03)	1.49 (0.4)
Design D2	664.65 (12.42)	1.21 (0.006)	4.8 (10.12)	100 (0)	1.5 (0.4)

Table 4.1.: Summary of centralized results for tariff T1. Columns refer to the economic and technical KPIs, whereas rows refer to the two market designs and the individual benchmark. The table reports the means of the KPIs for 10 days of high and low PV generation respectively, whereas standard deviations are depicted between parentheses.

	REC Bill [€]	PAR+	PAR-	SCR [%]	SSR [%]
Day			High PV		
Benchmark	171.28 (6.66)	2.37 (0.03)	4.41 (0.14)	73.08 (1.4)	47.49 (0.78)
Design D1	171.28 (6.66)	2.4 (0.03)	4.18 (0.35)	73.08 (1.4)	47.49 (0.78)
Design D2	161.95 (6.85)	2.39 (0.04)	3.25 (0.34)	83.37 (0.02)	54.18 (1.02)
Day			Low PV		
Benchmark	362.96 (4.67)	2.22 (0.01)	4.8 (10.12)	99.99 (0.03)	1.49 (0.4)
Design D1	362.96 (4.67)	2.22 (0.01)	4.8 (10.12)	99.99 (0.03)	1.49 (0.4)
Design D2	362.96 (4.67)	2.22 (0.01)	4.8 (10.12)	100 (0)	1.49 (0.4)

Table 4.2.: Summary of centralized results for tariff T2. Columns refer to the economic and technical KPIs, whereas rows refer to the two market designs and the individual benchmark. The table reports the means of the KPIs for 10 days of high and low PV generation respectively, whereas standard deviations are depicted between parentheses.

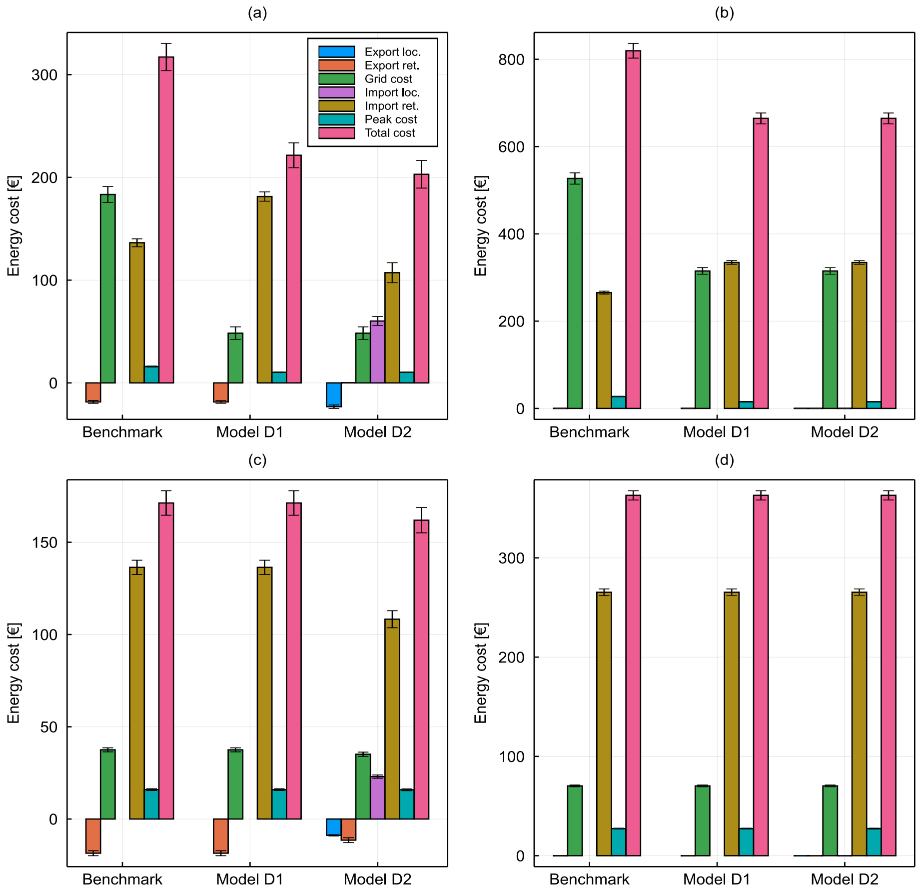


Figure 4.3.: Mean (and standard deviation) of the components of the total REC costs with T1 tariff, for the individual benchmark, D1 and D2, (a) for the high PV days and (b) the low PV days. The subplots (c) and (d) represent the T2 tariff case with $\gamma = 0.5$, corresponding to days with high PV and low PV respectively.

We first analyze the results of the quadratic grid tariff design in Table 4.1. Compared to the benchmark solution, total costs are on average 30.14% (18.9%) lower for high (low) PV days under REC design D1. The gain mainly originates in lower upstream grid costs and to a lesser extent individual peak costs, which is confirmed by the lower PAR values. Extra savings of 8.36% are obtained with the D2 design compared to D1, for high PV days.

According to the data from Table 4.2 and Fig.reffig-2(c)-(d), we can assert that the benchmark and model D1 are equivalent to the linear grid tariff design. We observe a cost decrease of 5.45% for the D2 model in the high PV case. Note the smaller difference in this case than with the T1 tariff.

The REC with mutualization of local excess generation (i.e., model D2) leads to the lowest total cost, and to the best technical KPI values, compared with the benchmark and D1 models. Indeed, the excess of local production is exchanged inside the REC at more advantageous prices than retail market prices, reducing commodity costs and the total REC bill. Sharing the surplus produced locally increases the incentive for members to coordinate their efforts, thereby increasing their self-consumption and self-sufficiency. Note that total bills obtained for tariffs T1 and T2 should not be compared on an absolute basis: our results aim to show that our modeling framework can be adapted to different grid tariff structures, but the study of the ideal grid tariff structure for communities from the regulatory point of view is out of the scope of this work.

For centralized models, the individual member bills are obtained ex post: the optimal total costs are distributed according to the billing mechanisms [EB,NET,VCG,CB] described in Section 4.3.2. In addition, we show that the D2 solution Pareto-dominates the D1 solution in [EB,NET] methods for both T1 and T2, since some members are better off without making another worse off. For the sake of conciseness, we don't include the quantitative results supporting that claim here. In the case of continuous billing, this is also true for T2 grid pricing and for the high PV case with T1 pricing, whereas the difference is negligible for low PV. Note that the [CB] individual bill for a member of the D1 model is identical to the one obtained if he had remained alone in the Benchmark.

4.6.2. Noncooperative games results

We focus on the numerical resolution of NEPs (4.25) for model D1 and GNEPs (4.27) for model D2. For each game, we start by calculating the REC global costs and inefficiency of equilibria compared with social optimum. Next, we look at members' bills and its variations from the centralized case. A summary of all the main empirical findings can be found in Table 4.6.

Design D1 as a noncooperative game

a) *For design D1 with tariff T1, the inefficiencies of the computed equilibria compared to the social optimum are zero for billings [EB, NET, VCG], and remain very limited for billing [CB].*

We first study the total REC costs and technical KPIs for the four cost-distribution methods proposed in the noncooperative day-ahead scheduling problems (4.25) under T1 pricing only, since T2 tariff is not relevant in this case. Table 4.3 summarises the mean and standard deviations of the results. The longest simulation times are 780s for [EB,NET,CB] and 1200s for [VCG]. Note that if the initial values correspond to a social optimum, then Algorithm 2 converges after 2 iterations in a few seconds for [EB,NET,VCG].

	REC Bill [€]	PAR+	PAR-	SCR [%]	SSR [%]
Day			High PV		
EB	221.55 (12.12)	1.19 (0.03)	2.45 (0.07)	73.08 (1.4)	47.49 (0.78)
NET	221.55 (12.12)	1.19 (0.03)	2.45 (0.07)	73.08 (1.4)	47.49 (0.78)
VCG	221.55 (12.12)	1.18 (0.03)	2.45 (0.07)	73.08 (1.4)	47.49 (0.78)
CB	224.85 (11.92)	1.43 (0.02)	2.42 (0.07)	73.08 (1.4)	47.49 (0.78)
Day			Low PV		
EB	664.65 (12.42)	1.21 (0.006)	4.8 (10.12)	99.99 (0.03)	1.49 (0.4)
NET	664.65 (12.42)	1.21 (0.004)	4.8 (10.12)	99.99 (0.03)	1.49 (0.4)
VCG	664.65 (12.42)	1.21 (0.005)	4.8 (10.12)	99.99 (0.03)	1.49 (0.4)
CB	669.54 (12.41)	1.39 (0.003)	4.8 (10.12)	99.99 (0.03)	1.49 (0.4)

Table 4.3.: Summary of decentralized results for model D1 with tariff T1.

As predicted by Theorem 4.3 and Corollary 4.1, the REC total costs at the computed equilibrium correspond to the social optimum of P_1 (221.55€) for the daily allocation keys [EB,NET,VCG]. Then, these billing mechanisms are efficient (PoA=1). In addition, the standard deviations are identical.

The [CB] billing, on the other hand, leads to a sub-optimal solution. We quantify the inefficiency as $(\sum_{i\in\mathcal{N}} b_i^{\mathrm{CB1}}(\Theta^*) - C^*_{\mathcal{N}})/C^*_{\mathcal{N}}$, where Θ^* is a NE and $C^*_{\mathcal{N}}$ the social optimum. The [CB] inefficiency remains however small: we obtain a mean inefficiency of 1.49% for the high PV days, and 0.73% on the low PV days. Thus, we have the following bound on the PoA of the game with [CB]:

$$\mathrm{PoA}(G) \geqslant \frac{224.85}{221.55} \approx 1.0149.$$

In fact, the upstream grid contribution of (4.24a) is subject to strategy in this billing, explaining the greater inefficiency during high PV days. The PARs differ from the daily billings, whereas SCR and SSR remain the same.

***b)** For design D1 with tariff T1, the individual bills obtained with the game formulation and via the ex-post distribution of the social optimum are equal to billings [EB, NET, VCG], and we observe non-negligible deviations for a limited number of individuals for [CB].*

It is important to note that, while the daily cost distribution approaches tend to minimize the total REC costs, some users may experience lower profits. Figure 4.4 depicts the individual invoices of the 55 members set, at the computed Nash equilibrium on high PV days.

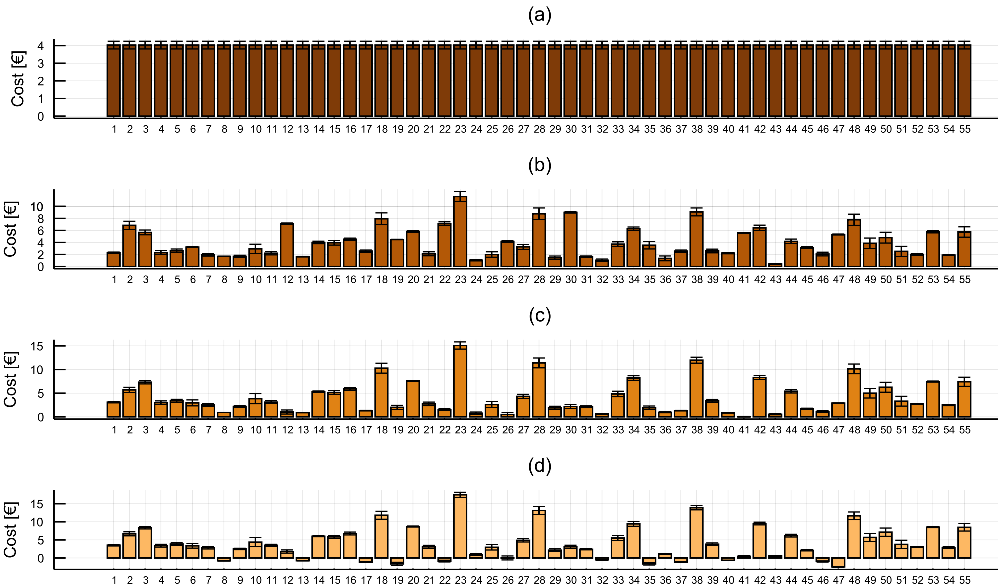


Figure 4.4.: Mean (and standard deviation) individual bills for the NEP (4.25) under tariff T1 with (a) [EB], (b) [NET], (c) [VCG] and (d) [CB] on the High PV days.

Overall, it appears that no clear trend can be observed regarding the allocation method, which is the most suited to members. It is, however, not the aim of this chapter to study the problem of fairness of allocation methods (we refer to [76, 72, 60, 127] for that purpose): our purpose is to compare individual bills when adopting the centralized and decentralized approaches, with the allocation method as a parameter.

Results showed that individual bills obtained with the centralized and decentralized approaches are overall equivalent, which validates Theorem 4.3.1. Although it is clear for [EB, NET, VCG], this is not so straightforward for the continuous billing [CB]. Figure 4.5 shows, for each member, the average percentage of changes between the bill b_i^{CB1} and the one obtained after ex-post allocation of the social optimum.

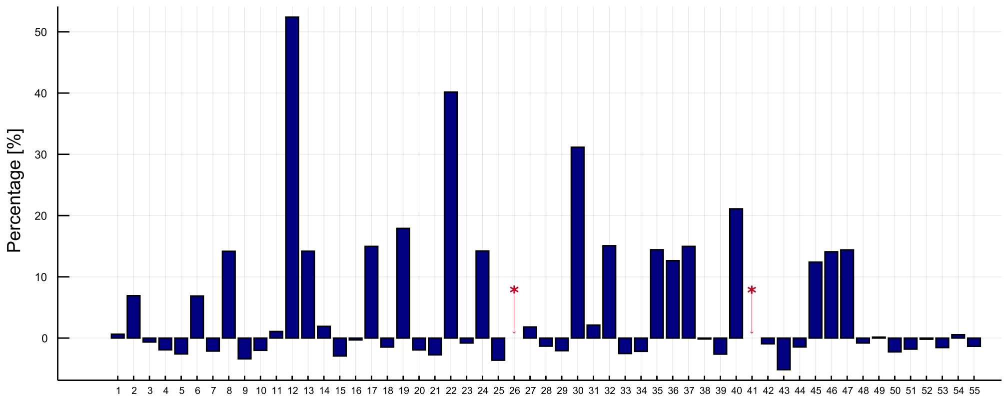


Figure 4.5.: Mean percentage of changes between individual bills b_i^{CB1} and the one obtained from the centralized method with [CB] on high PV days. Data for players 26 and 41 are not displayed due to scale considerations.

The relative difference reaches 4622.5% for member 41. This outlier is explained by the fact that, for one of the days in our use case, the player goes from a bill very close to zero at the social optimum, to 0.384 €. We observe a value of 84% for player 26, who is, in fact, a pathological case like players 12, 22 and 40. In fact, their bills are lower than 1 in absolute value, which explains the large relative change (see Fig.4.4(d)). For all members of the REC, the mean difference in absolute number does not exceed 0.72 €. This value is associated to member 30, which so corresponds to the largest deviation (31.17%) excluding the pathological cases mentioned. Therefore, we conclude that, in the D1 case, the daily costs distribution among members are equals when solving the optimization problem (4.19) and the game (4.25) for [EB, NET, VCG]. The differences are however not negligible for the continuous case, for some of the members.

Design D2 as a noncooperative game

We proceed in a similar way for the noncooperative scheduling problem with local exchanges (4.27), first for the grid tariff structure T1 and then T2 pricing.

***c)** For design D2 and tariff T1, the inefficiency of the computed equilibria compared to the social optimum is zero for billing [EB], is negligible for billings [NET, VCG], and remains very limited for billing [CB].*

We first examine the problem from a macroscopic perspective for the four cost allocation methods. Table 4.4 summarises the mean and standard deviations of the results of the GNEP resolution. If the set of initial values corresponds to a social optimum, the worst simulation time is 83s for [EB], 107s for [NET, VCG] and 149s for [CB].

	REC Bill [€]	PAR+	PAR-	SCR [%]	SSR [%]
Day			High PV		
EB	203.02 (13.42)	1.19 (0.032)	2.46 (0.07)	100 (0)	65.02 (2.3)
NET	203.04 (13.42)	1.19 (0.032)	2.46 (0.07)	100 (0)	65.02 (2.3)
VCG	203.05 (13.42)	1.19 (0.032)	2.46 (0.07)	100 (0)	65.02 (2.3)
CB	203.83 (13.6)	1.24 (0.076)	2.45 (0.07)	100 (0)	65.02 (2.3)
Day			Low PV		
EB	664.65 (12.42)	1.21 (0.006)	4.8 (10.12)	100 (0)	1.49 (0.4)
NET	664.67 (12.42)	1.21 (0.006)	4.8 (10.12)	100 (0)	1.49 (0.4)
VCG	664.68 (12.42)	1.21 (0.006)	4.8 (10.12)	100 (0)	1.49 (0.4)
CB	668.05 (13.13)	1.36 (0.02)	4.8 (10.12)	100 (0)	1.49 (0.4)

Table 4.4.: Summary of decentralized results for model D2 with tariff T1.

For [EB] billing, the computed VE is a social optimum for problem P_2 (4.20), because the same value of total costs can be observed in Table 4.1 and 4.4, for both high PV (203.02€) and low PV (664.65€). This corroborates Theorem 4.5.2 and Corollary 4.2.2. We observe that the total bill does not correspond exactly to the social optimum for the [NET] and [VCG] billings. As explained in Section 4.4.3, although Theorem 4.5.1 assures that there are GNEs that are socially optimal, we are not able to prove that the PDA converges to one of them. As expected, [CB] leads to a sub-optimal solution. The inefficiencies (i.e., deviations from the social optimum) are calculated as $(\sum_{i\in\mathcal{N}} b_i^* - C_{\mathcal{N}}^*)/C_{\mathcal{N}}^*$, where $C_{\mathcal{N}}^*$ is the social optimum. We observe in Table 4.6, that the inefficiencies remain, however, very small, especially for [NET,VCG] (0.007% and 0.014% respectively, contrarily to 0.4% for [CB]). Thus, we have the following bound on the PoA of the game with [CB]:

$$\mathrm{PoA}(G) \geqslant 1.005.$$

Note that the inefficiency of [CB] in D2 is lower than for D1.

d) *For design D2 with tariff T1, the individual bills obtained with the game formulation and via the ex-post distribution of the social optimum are equal for billing [EB]. The difference is negligible for billings [NET, VCG], and we observe non negligible deviations (although smaller than for D1) for a limited number of individuals for [CB].*

Figure 4.6 shows the invoices of the 55 members at the computed equilibrium on high PV days for the grid tariff T1. In a similar way to the centralized case, we note that the D2 solution Pareto-dominates D1 solution with [EB,NET] distributions (see Fig.4.4(a)-(b) and Fig.4.6(a)-(b)).

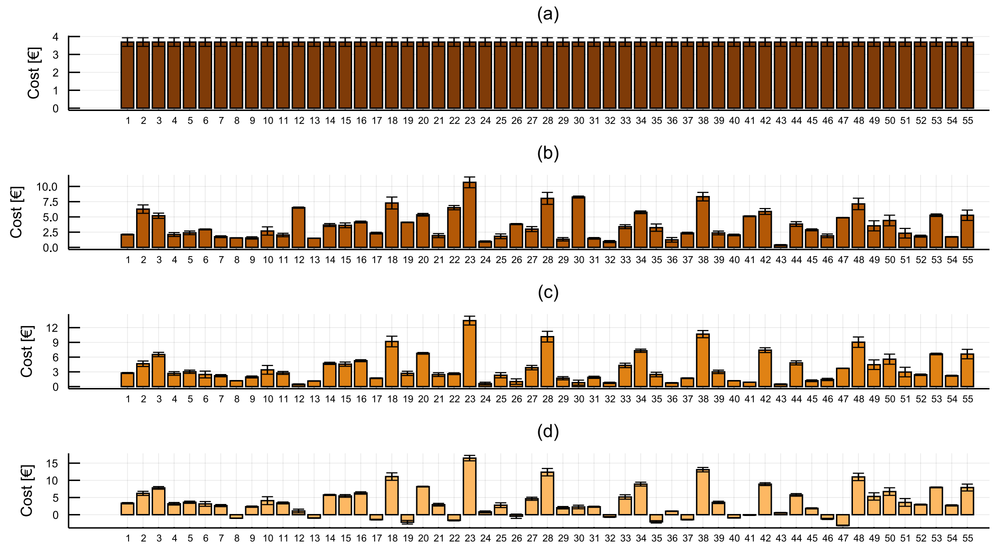


Figure 4.6.: Mean (and standard deviation) individual bills for the GNEP (4.27) under tariff T1 with (a) [EB], (b) [NET], (c) [VCG] and (d) [CB] on the high PV days.

The individual bill derived from the [EB] mechanism is equal to the one obtained ex-post in the centralized case. The individual changes under [NET,VCG,CB] methods are displayed in Figure 4.7.

As we can see on Fig. 4.7(a), the percentages are quite small for the [NET] method. The percentage maximum reaching 0.022%, for member 17, and a difference of 0.001€ (this may correspond to a numerical or rounding error). The same can be said of the [VCG] billing, as shown in Fig.4.7(b), with the percentage maximum at 0.21% and a difference of 0.006€. Once again, we

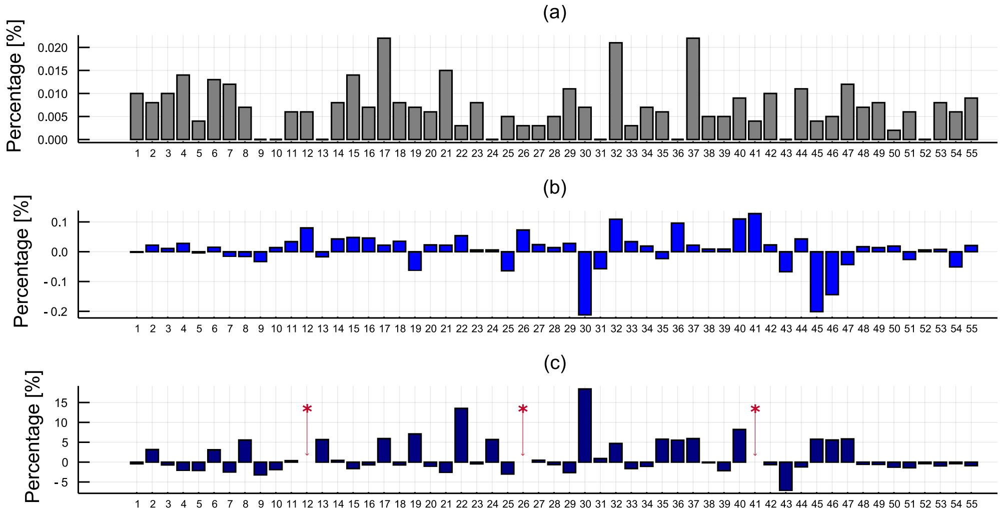


Figure 4.7.: Mean percentage of changes between individual bills and the one obtained from the centralized method with (a) [NET], (b) [VCG] and (c) [CB] on high PV days under tariff T1. Data for players 12, 26 and 41 are not displayed due to scale considerations.

come across pathological cases with higher percentages for the [CB] mechanism (see Fig.4.7(c)). On one day, member 26's bill rises from 0€ to 0.064€. This explains his percentage of 2131.94%. We observe a value of 47% and 65% for players 12 and 41 respectively, but their bills are lower than 1 in absolute value, which explains the large relative change (see Fig.4.6(d)). In fact, for all members of the REC, the mean difference in absolute number does not exceed 0.4€ in the [CB] distributions. Excluding these special cases, the biggest difference is 18.4% corresponding to member 30. For daily billing mechanisms, the individual bills are the same whether the optimization problem (4.20) or the game (4.27), under the grid tariff T1, is solved. The differences remain acceptable for the continuous case and are better than in model D1.

e) *For design D2 with tariff T2, the inefficiencies of the computed equilibria compared to the social optimum are zero for billing [EB,CB], and are negligible for billings [NET, VCG].*

We display results for tariff T2, inspired by Belgian regulation: we apply a discount on grid fees for energy consumed locally (as in Brussels), in this case 50%, and include a peak component in the grid invoice (as in Flanders). We begin by comparing the average total costs of the community obtained from the GNEP resolution at Table 4.5. The longest simulation times are 83s for [EB], 103s for [NET,VCG] and 80.6s for [CB], when the set of initial values is a social optimum.

	REC Bill [€]	PAR+	PAR-	SCR [%]	SSR [%]
Day			High PV		
EB	161.95 (6.85)	2.39 (0.04)	3.3 (0.31)	83.38 (1.6)	54.18 (1.03)
NET	161.96 (6.85)	2.39 (0.04)	3.3 (0.31)	83.38 (1.6)	54.18 (1.03)
VCG	161.97 (6.85)	2.39 (0.04)	3.3 (0.31)	83.38 (1.6)	54.18 (1.03)
CB	161.95 (6.85)	2.39 (0.04)	3.3 (0.31)	83.38 (1.6)	54.18 (1.03)
Day			Low PV		
EB	362.96 (4.67)	2.22 (0.006)	4.8 (10.12)	100 (0)	1.49 (0.4)
NET	362.98 (4.67)	2.22 (0.006)	3.9 (0.96)	100 (0)	1.49 (0.4)
VCG	362.99 (4.67)	2.22 (0.006)	3.8 (0.75)	100 (0)	1.49 (0.4)
CB	362.96 (4.67)	2.22 (0.006)	4.8 (10.12)	100 (0)	1.49 (0.4)

Table 4.5.: Summary of decentralized results for model D2 with tariff T2 and $\gamma = 0.5$.

The VE corresponds to a social optimum of P_2 problem (4.20) for [EB,CB] allocation methods as stated by Theorem 4.5.2 and 4.5.4, as well as Corollary 4.2.2 and 4.2.3. As with T1 pricing, we are unable to prove that the PDA converges towards a social optimum for [NET,VCG] billings. However, their inefficiencies are so small that they can be considered negligible (see Table 4.6). We can therefore say that the calculated VEs of GNEP (4.27) are equivalent to the social optimum of P_2 problem (4.20).

f) *For design D2 with tariff T2, the individual bills obtained with the game formulation and via the ex-post distribution of the social optimum are equal for billing [EB]. The difference is negligible for billings [NET, VCG], and remains very limited for [CB] for a few individuals.*

Figure 4.8 shows the 55 members' bills at the computed VE on high PV days for the grid tariff T2 and a discount factor $\gamma = 0.5$.

The [EB] billing method induced the same individual costs for centralized and decentralized systems. This is not necessarily the case for the continuous method. Theorem 4.5.4 does not guarantee equivalence at the individual bill level. We need to study the empirical individual changes result of our use-case, shown in Figure 4.9.

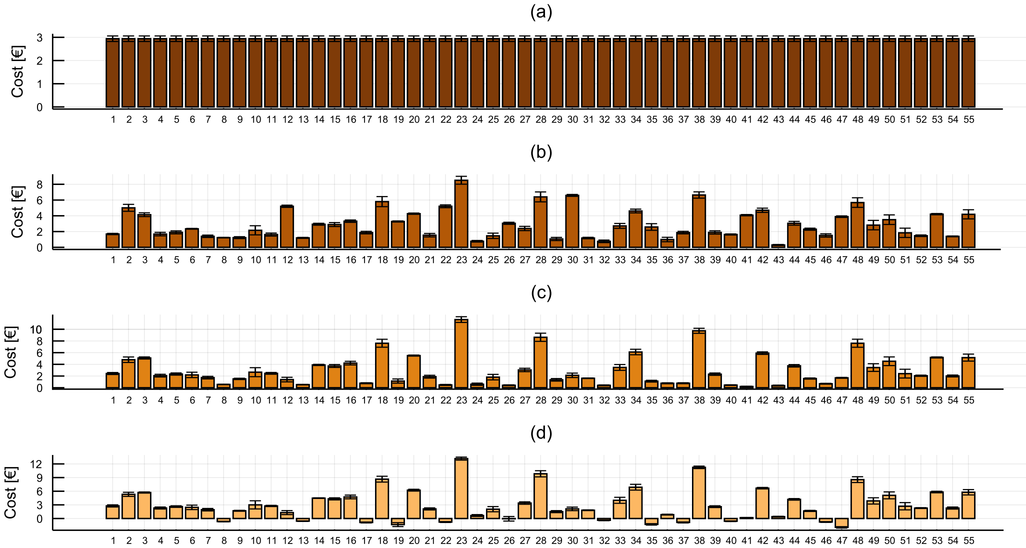


Figure 4.8.: Mean (and standard deviation) individual bills for the GNEP (4.27), under tariff T2 and $\gamma = 0.5$, with (a) [EB], (b) [NET], (c) [VCG] and (d) [CB] on the high PV days.

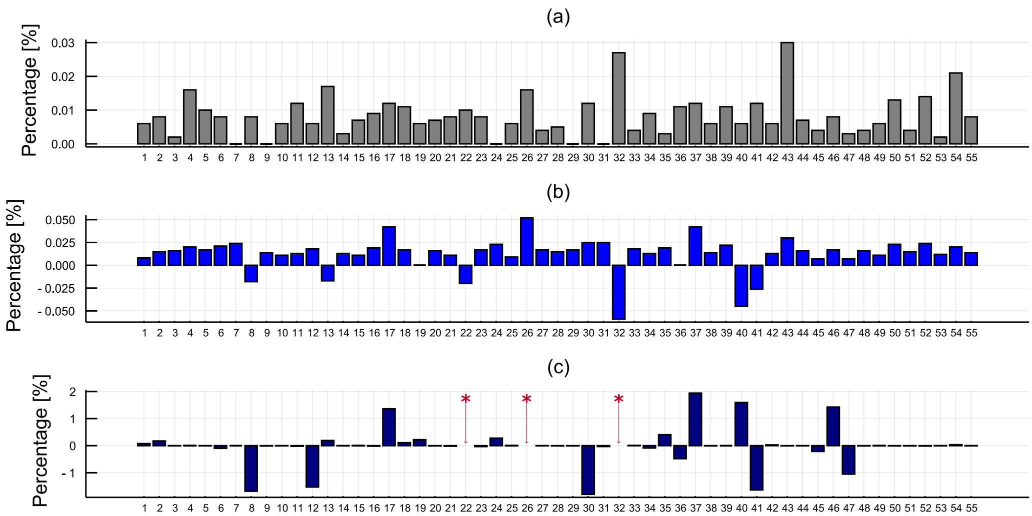


Figure 4.9.: Mean percentage of changes between individual bills and the one obtained from the centralized method with (a) [NET], (b) [VCG] and (c) [CB] on high PV days under tariff T2 with $\gamma = 0.5$. Data for players 22, 26 and 32 are not displayed due to scale considerations.

A view of the Figure 4.9(a)-(b) allows us to draw similar conclusions to tariff T1 for [NET,VCG] cost distributions. We focus on the continuous method (see Fig.4.9(c)). The missing data on the graph are 5.18%, 6% and 3.47% associated with members 22, 26 and 32, respectively. Counting player 37, these are once again users whose invoices are smaller than 1 in absolute value, which explains the slightly higher percentages than the others (see Fig.4.8(d)). The difference in absolute number does not exceed 0.150€. Excluding these special cases, the biggest percentage is 1.8%, associated with member 30. These gaps are relatively small and can be considered negligible in the context of energy costs. So, replacing centralized with decentralized maintains the same values of individual invoices for each allocation method examined, with the grid tariff T2. Note that the same conclusions apply to low production days.

Summary of analytical and empirical results for noncooperative games

Table 4.6 summarizes the theoretical and empirical results related to the noncooperative game models, for the different cost allocation methods and for the two grid tariff structures. Columns aim to answer 5 theoretical and practical questions:

- Q1 (Existence): does a (G)NEP exist?
- Q2 (Efficiency of the equilibrium): is there an equilibrium which is also a social optimum?
- Q3 (Individual deviations): is there an equilibrium for which the individual bills are equivalent to the one computed ex-post in the centralized case?
- Q4 (Resolution): Does the Proximal Decomposition Algorithm (PDA) converge towards an equilibrium?
- Q5 (Empirical verification): What is the average computed inefficiency in terms of total REC costs for our use-case, and what are the worst computed individual deviations?

More precisely, columns Q1 to Q4 report the analytical conclusions of Sections 4.4.2-4.4.3 and the Appendices, and column Q5 reports the main findings of section 4.6. Note that in Table 4.6, the symbol "?" indicates cases for which we have no conclusive theoretical results. This underlines the uncertainty or absence of formal guarantees concerning these situations. Overall, for the two proposed local market designs, and for the four studied billings and two grid tariff schemes, we outline two important conclusions of practical significance for the management and billings of Renewable Energy Communities:

1. The total REC costs obtained with the centralized optimization-based and noncooperative game formulations are either identical, or differ slightly

			Q1	Q2	Q3	Q4	Q5	
							REC [%]	Worst Ind.[%]
Linear T2	D2	EB	Yes (Thm.4.4)	Yes (Thm.4.5.2 & Cor.4.2.2)		Yes (Appendix B.3)	0 (4.6.2e))	0 (4.6.2f))
		NET		Yes (Thm.4.5.1 & Cor.4.2.1)			0.008 (")	0.03 (")
		VCG					0.014 (")	0.052 (")
		CB		Yes (Thm.4.5.4 & Cor. 4.2.3)			0 (")	1.8* (")
Quad. T1	D1	EB	Yes (Thm.4.2)	Yes (Thm.4.3.1 & Cor. 4.1)		Yes (Appendix A.2)	0 (4.6.2a))	0 (4.6.2b))
		NET						
		VCG						
		CB		? (Thm.4.3.2)		Yes (Appendix A.2)	1.49 (")	31.17* (")
	D2	EB	Yes (Thm.4.4)	Yes (Thm.4.5.2 & Cor. 4.2.2)		Yes (Appendix B.3)	0 (4.6.2c))	0 (4.6.2d))
		NET		Yes (Thm.4.5.1 & Cor. 4.2.1)		Appendix B.3 (Rem. B.1)	0.007 (")	0.022 (")
		VCG					0.014 (")	0.21 (")
		CB		? (Thm.4.5.3)		Yes (Appendix B.3)	0.4 (")	18.4* (")

Table 4.6.: Summary of analytical and empirical results of the decentralized approach.

(with a negligible difference). This means that, at the macroscopic level, any study which aims at quantifying the economic impact of a massive roll-out of RECs in the electricity system can rely on the light and fast centralized formulation.

2. The individual bills of the REC members are either identical or differ very slightly when computed ex-post from the results of the centralized optimization-based model, compared to the noncooperative game resolution, apart from the [CB] billing scheme with quadratic grid tariff T1, where non-negligible deviations occur. This means that, if we except the [CB] case with tariff T1 and if sharing members data with the Community Manager is not an issue (if the latter issues a problem, a PDA approach can be employed and always converge towards an equilibrium), the individual bills can be obtained with the fast centralized formulation, which is an important information for community managers for billing purposes.

4.7. Sensitivity analysis on retail electricity prices

In this section, we investigate the influence of retail electricity price on the operation of the REC with D2 design. To this end, we carry out a sensitivity analysis of retail electricity prices. We study a REC composed of 25 members included in the community of 55 participants studied in Sections 4.5 and 4.6. The day-ahead energy scheduling model is run for 10 days with high PV production. The value of λ_{imp} varies from 0.06 to 0.16 €/kWh with steps of 0.01. Then, we set all other prices over the full horizon: $\lambda_{iloc} = 0.13$ €/kWh,

	PV	ESS (capacity, max. power)	Total consumption	Flexibility level
User 8	9 kWp	(0 kWh, 0 kW)	37.36 kWh	0%
User 14	3 kWp	(0 kWh, 0 kW)	116.03 kWh	36.38%
User 21	9 kWp	(14 kWh, 5 kW)	11.84 kWh	138.24%
User 22	8 kWp	(14 kWh, 5 kW)	82.21 kWh	78.1%

Table 4.7.: Characteristics of the 4 selected end-users.

$\lambda_{eloc} = 0.05$ €/kWh, $\lambda_{exp} = 0.04$ €/kWh and $\beta = 0.11$ €/kW.

In addition to the total REC costs, we study individual member invoices according to five cost distribution methods. These are [NET, VCG, CB] as presented in Section 4.3.2, as well as the classic versions of [NET, VCG], studied in [76]. Named [Clas. NET] and [Clas. VCG], these cost allocations correspond to the equations (4.22) and (4.23) respectively, but without the absolute values. Figure 4.10-4.12 depicts the individual invoices of 4 users selected from the 25 members set. Table 4.7 details their electrical equipment, daily energy needs and flexibility level (computed as the ratio between the available flexibility in kWh, including storage, and the total consumption over the period).

4.7.1. Grid tariff structure T1

We first study the total REC costs and individual bills for the five cost allocations with T1 grid tariff design and $\alpha = 0.00109488$ €/kWh^2.

Total REC costs

The Fig. 4.10(a) depicts the mean REC total cost composition of the global optimization (4.20) as a function of the import retail fee. Despite numerical errors (max 0.04%), the global optimization model (4.20) and the decentralized model (4.27) share the same trends in terms of REC total cost. Therefore, we do not illustrate REC bill in Fig. 4.11 and the Fig. 4.10(a) analysis concerns both formulations.

The total cost increases linearly for a value less than or equal to 0.11 €/kWh. In this case, the import retail fee is lower than the local import price and no agent will buy from the community, so the selling members sell their surplus energy on the classical market. There is therefore no exchange inside the REC. There is a behavior change when $\lambda_{imp} = 0.12$ €/kWh although it is still lower than $\lambda_{iloc} = 0.13$ €/kWh. In fact, for this value, the difference between the price of import and export in the retail market is equal to that in the community.

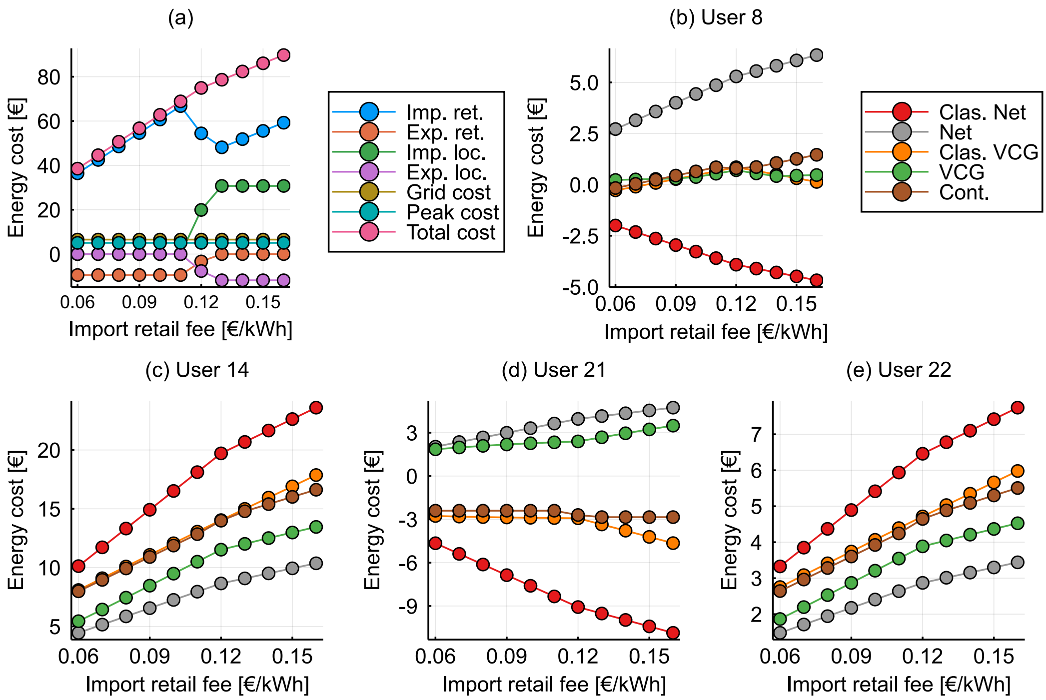


Figure 4.10.: Design T1: Mean total cost composition of the centralized REC in (4.20) (a) and each subplot (b)-(e) represents the different personal bills of a player in Tab. 4.7, as a function of λ_{imp}.

Members will therefore play on both sides. Finally, with a larger import retail and so $\lambda_{iloc} - \lambda_{eloc} < \lambda_{imp} - \lambda_{exp}$, users are more likely to buy energy from the REC pool. It is therefore more profitable for sellers of excess energy to make available as much as possible to other members. As a result, the total cost increase is smaller than for the lower values. As a summary, as long as

$$\lambda_{iloc} < \lambda_{imp} \text{ or } \lambda_{exp} < \lambda_{eloc}, \tag{4.36}$$

$$\lambda_{iloc} - \lambda_{eloc} < \lambda_{imp} - \lambda_{exp}, \tag{4.37}$$

the energy is exchanged inside the community at more advantageous prices than the retail market prices. This reduces commodity costs and as a consequence the total bill.

Individual bills

Figures 4.10(b)-(e) and 4.11 show the individual bills of the 4 members of Table 4.7, at the computed optimal solution and Nash equilibrium, respectively.

Note that [Clas. NET, Clas. VCG] billing methods are not calculated in the game formulation (see Fig. 4.11), because of the non-convexity they can cause. Despite numerical errors, for each member and cost distribution, the global optimization model (4.20) and the decentralized model (4.27) share the same trends.

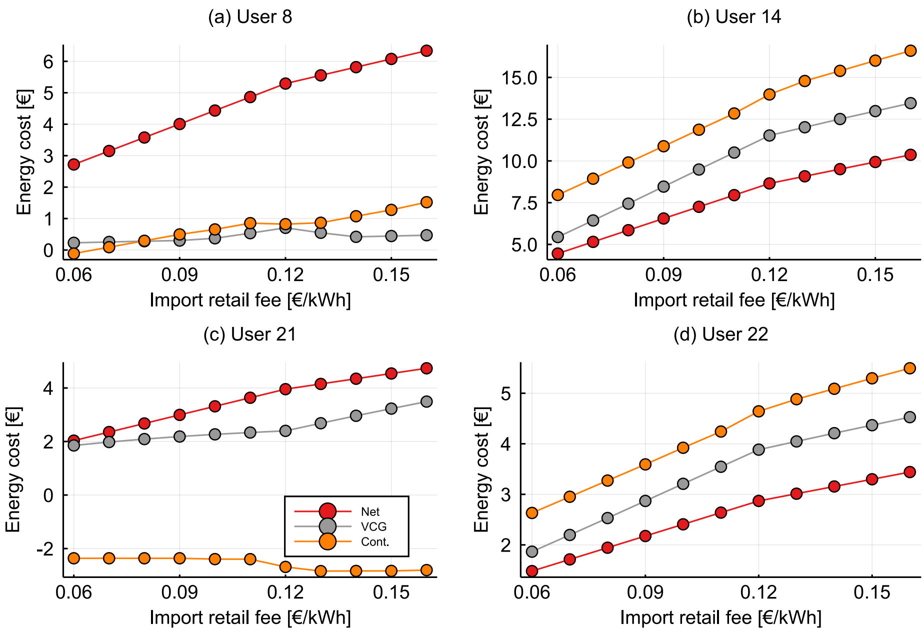


Figure 4.11.: Design T1: Mean cost allocation of the GNEP (4.27), each subplot (a)-(d) represents the personal bills of a player in Tab. 4.7, as a function of λ_{imp}.

Each billing policy has its own specificities. As noted in [71], the VCG schemes depend on the cost structure and the individual profiles. Some prosumers, such as user 21, have a negative relative contribution, indicating, in fact, a positive impact of the member on the total costs. In the case of [VCG], the distribution key is positive, leading to an increase in the denominator. Thus, users 14 and 22 have lower costs compared to the [Clas. VCG, CB], but a higher bill than [NET]. The latter presents a similar phenomenon, but much more amplified. The [NET] method incentivizes members to minimize interactions with the grid, so user 21 is strongly impacted. We also notice that [NET] tends to be egalitarian, unlike the [Clas. NET], in this market design. Finally, we see that [CB] provides negotiating power to members: outcomes for users with PV but no storage and flexibility assets tend to degrade compared to VCG schemes.

4.7.2. Grid tariff structure T2

We display results for tariff T2 with $\alpha = 0.027$ €/kWh, both without ($\gamma = 1$, Fig. 4.12) and with a discount on tariff grid for electricity consumed locally ($\gamma = 0.5$, Fig. 4.13). Thanks to this linear structure, we also establish a comparison benchmark in which every user optimizes his individual electricity bill, without any community operation.

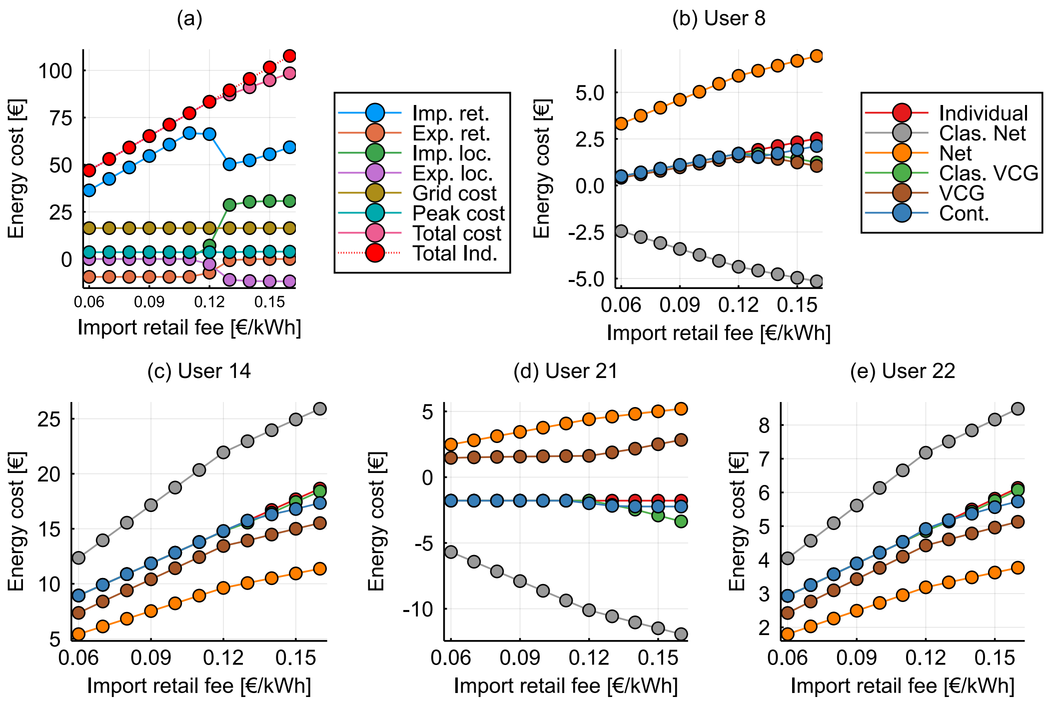


Figure 4.12.: Design T2: Mean total cost composition of the REC with $\gamma = 1$ and the sum of the individual bills in the benchmark (a). Each subplot (b)-(e) represents the different personal bills of a player in Tab. 4.7.

Total REC costs

Fig. 4.12(a) shows the mean REC total cost composition with $\gamma = 1$ and the sum of the individual bills in the benchmark, according to the import retail fee. The $\gamma = 0.5$ case is illustrated in Fig. 4.13. It is interesting to note that the change of attitude does not occur at the same time or with the same intensity depending on the discount granted. Indeed, members are starting to benefit from internal exchanges once

$$\lambda_{iloc} - \lambda_{eloc} \leqslant \lambda_{imp} + \alpha(1-\gamma) - \lambda_{exp}. \quad (4.38)$$

However, for $\gamma = 1$, Figure 4.12(a) shows that members share their trade between the REC pool and the retail market until 0.14 €/kWh, even though both

$$\lambda_{iloc} < \lambda_{imp},$$
$$\lambda_{iloc} - \lambda_{eloc} < \lambda_{imp} - \lambda_{exp},$$

are verified. Actually, they prioritize the community as soon as

$$\lambda_{iloc} - \lambda_{eloc} + \alpha \leqslant \lambda_{imp} - \lambda_{exp}, \quad (4.39)$$

i.e, for $\lambda_{imp} = 0.15$ €/kWh.

In the discount situation, i.e., when $\gamma = 0.5$, Figure 4.13 shows that it becomes profitable to trade in the community when $\lambda_{imp} = 0.11$, while continuing to sell on the retail market. Although $\lambda_{imp} = \lambda_{iloc}$ at 0.13 €/kWh, thanks to the reduction on the local grid, it is more profitable to exchange as much as possible in the REC pool, which leads to a larger reduction in total costs than in previous case.

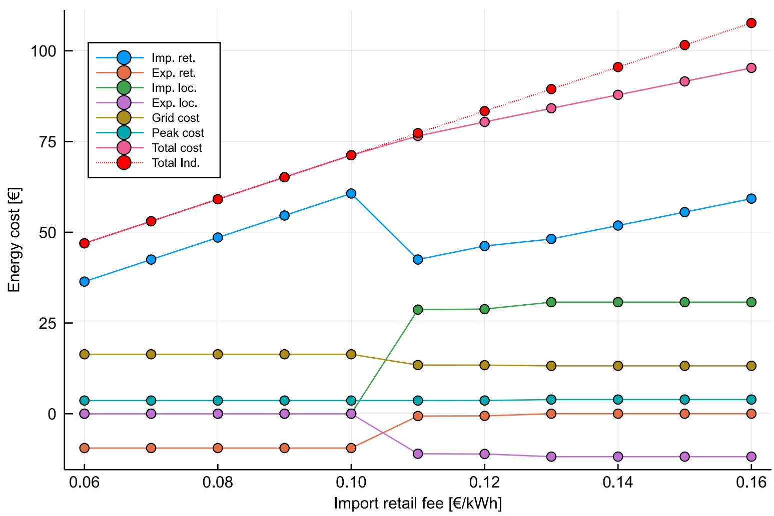


Figure 4.13.: Design T2: Mean total cost composition of the REC with $\gamma = 0.5$, and the sum of the individual bills in the benchmark.

	Scenario	1	2	3	4	5	6	7	8	9	10	11
	λ_{imp} value	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16
REC total costs	T1	38.54	44.6	50.67	56.74	62.81	68.88	74.95	78.65	82.35	86.06	89.76
	T2, $\gamma = 1$	46.96	53.03	59.1	65.17	71.24	77.3	83.37	87.24	91.05	94.76	98.47
	T2, $\gamma = 0.5$	46.96	53.03	59.1	65.17	71.24	76.53	80.39	84.16	87.87	91.57	95.27
Grid costs	T1	11.58	11.58	11.58	11.58	11.58	11.58	11.58	11.58	11.58	11.58	11.58
	T2, $\gamma = 1$	20.01	20.01	20.01	20.01	20.01	20.01	20.01	20.01	20.22	20.29	20.29
	T2, $\gamma = 0.5$	20.01	20.01	20.01	20.01	20.01	17.03	17.03	17.1	17.1	17.1	17.1

Table 4.8.: Mean total and grid costs summary.

Table 4.8 shows that T1 has the lowest total costs and grid costs, which remain constant despite the change in members' behavior. Nonetheless, a reduced tariff on the local grid (thus a smaller γ) increases the incentive for members to trade in the REC pool, which increases self-consumption, resulting in a decrease of the grid costs and so the total costs.

Members interest in joining the REC

Each subplot Figure 4.12(b)-(e) displays average individual bills. We seek to determine which distribution method incentivizes users to join the community rather than staying outside (see red lines). Despite differences, design T2 with $\gamma = 1$ and $\gamma = 0.5$ share trends in terms of cost allocation. Thus, they are not represented in Fig. 4.13 and the analysis concerns both formulations.

As we can see on Figure 4.12(b), the [NET] method damages the invoice of user 8. Users 14 and 22 have great interest in being part of the REC under both [NET,VCG] pricing methods. On the other hand, [Clas. NET] does not encourage them to follow recommendations or to be part of the community. Concerning [Clas. VCG, CB] billings, we observe bill reductions starting at $\lambda_{imp} = 0.13$; below this value, their preferences are neutral (Fig.4.12(c) and (e)). User 21 is a seller of surplus energy so the two new keys [NET, VCG] damage his bill, since its costs are positive, while the three other invoicing methods generate a profit. However, in [Clas. VCG, CB], it is the internal exchanges that will allow us to reduce the billing compared to the benchmark, whereas the [Clas. NET] billing really incentivizes the prosumer to participate in the community.

In brief, a community with grid tariff pricing T2 is beneficial to each user type whether the policy of cost distribution is [Clas. VCG] or [CB]. Other comments are similar to the analysis of individual bills in Section 4.7.1

4.8. Conclusion

This chapter compares two market designs for the optimal day-ahead scheduling of energy exchanges and members' appliances within renewable energy communities. The first one (D1) implements a collaborative demand-side management scheme inside a community where members' objectives are coupled through grid tariffs, the second (D2) allows the valuation of excess generation in the community and on the retail market. Two grid tariff structures are tested, an academic one which considers quadratic costs for the upstream grid contribution and one which reflects the Belgian regulations in terms of grid tariffs. Individuals' bills are obtained through 4 methods of cost allocation. Both models are formulated as optimization problems first, and as noncooperative games then. Analytical and empirical studies are conducted in order to compare D1 and D2, as well as the centralized and decentralized approaches. The models are tested on a use-case made of 55 members and compared with a benchmark situation where members act individually.

We first show that design D2 which includes the valuation of excess local generation, and to a lesser extent design D1, tends to lower the total REC electricity costs, and to improve the self-consumption, self-sufficiency and peak-to-average ratio of the community. For instance in high PV case, the model D1 saves an average of 30.14% on total costs for T1 pricing, while model D2 saves an extra 8.36% compared to D1. We observe a cost decrease of 5.45% for model D2 compared to the benchmark, with T2 pricing.

Then, we focus on the existence and the efficiency of the equilibria computed with the decentralized models. First, we show that there always exists an equilibrium that is a social optimum, with the three daily cost distributions [EB, NET, VCG]. This also holds for continuous billing [CB] if tariff T2 is in effect. Secondly, we show that the total REC costs obtained with the centralized and noncooperative game formulations are either identical, or differ very slightly (0.4% and 1.49% for [CB] with tariff T1 and designs D1 and D2 respectively). This means that, at the macroscopic level and with our hypotheses, any study which aims at quantifying the economic impact of a massive roll-out of RECs in the electricity system can rely on the light and fast centralized formulation.

Finally, we show, analytically when possible, and empirically if not, that members' individual bills obtained ex-post from the faster centralized model results and directly via decentralized approaches are exactly equivalent for the daily cost allocation methods [EB, NET, VCG], and very close for the [CB] method with T2 tariff (i.e., 1.8%), which is of practical significance for the community manager for billing purposes. The deviations for [CB] with tariff

T1 are not negligible, however (18 to 31% for some members).

In Section 4.7, we studied the impact of retail electricity prices and the two grid tariff structures on the operation of a REC with design D2. We demonstrate the existence of a threshold in the import retail price, depending on the difference between the import/export community prices and the import/export retail prices, for which the economic gains of operating as a REC increase significantly, for both grid tariffs T1 and T2. We also show that, according to our hypotheses (rational behavior of members), the realistic grid tariff design T2 is at least neutral or beneficial in terms of individual costs for each user type, provided that the cost allocation policy is [Clas. VCG] or [CB].

Apart from data privacy issues, the use of decentralized approaches for modeling communities remains of practical significance if community members pursue individual objectives of different natures (e.g., bill minimization for member 1, CO2 emissions minimization for member 2, etc.). Additionally, end-users may exhibit limited rationality in their decisions, and may not share the same risk attitudes. Further exploration of these aspects is kept for a future work.

Part II.

Extensive Games for New Member Integration with Investment

CHAPTER 5.

Extensive-Form Games and Prospect Theory

5.1. Introduction

Chapter 4 exposed the design of a low-voltage renewable energy community. We implemented a collaborative demand-side management scheme inside a community where members' objectives are coupled through grid tariffs, and allowing the excess generation to be shared among community members. We have extensively studied the day-ahead resources scheduling within such communities. We have proved that it is always possible to find a Nash equilibrium which leads to minimizing the social cost.

However, strategic games are mainly static, meaning that players choose their strategies independently and simultaneously, as mentioned in Section 3.2. So, these games do not integrate any notion of order and timing into the players' strategic decision-making, which is an important feature of many economic and industrial settings. For instance, a consumer responding to a price signal from the supplier, or the new member integration problem inside a REC (see Section 6.2). This raises the problem of representing and analyzing dynamic strategic environments. An *extensive-form game* (or an *extensive game*) is a detailed representation of the *sequential structure* of the decision-making process faced by players in a strategic situation [7]. Extensive games highlight the order in which players move, as well as the information available to the players at each stage of their choice process. An insight is provided in Section 5.2.

Classical theory focuses on the question: *how will rational players play?* As a reminder, game theory is based on the fundamental assumption that players are rational. This means that all players will choose the options that maximize their gains or minimize their costs. As explained in Chapter 2, the electrical system is tending to evolve towards a prosumer-centric system. Apart from technical specifications of the smart grid, important behavioral factors will determine users' decisions. Providing members with the opportunity to perform optimally is no guarantee that they will follow these recommendations. End-user behavior and preferences are therefore central side to this realization. It is thus important to consider models where the bounded rationality of the end-users is taken into account. Section 5.3 presents the descriptive decision-making model proposed by Kahneman and Tversky in [77, 81], which aims to integrate insights from psychology to get better answers to the question: *how do humans play?*

5.2. Extensive-form games

The extensive form of a game is a complete description of 1) the set of players, 2) who moves when and what their choices are, 3) what players know when they move and 4) the players' payoffs as a function of the choices that are made. Von Neumann and Morgenstern provided a set-theoretic description of this model [8], but this section is mainly based on [7, 132].
We are particularly interested in the case of extensive games with *perfect information.* There is perfect information in such a game if each player, when making any decision, is perfectly informed of all the events that have previously occurred. Chess is a sequential game with perfect information.

5.2.1. Extensive game with perfect information

We provide a formal definition of an extensive game [7]. Note that actions ($x \in X := \prod_{i \in \mathcal{N}} X_i$) and strategies ($s \in \mathcal{S} := \prod_{i \in \mathcal{N}} \mathcal{S}_i$) are the same in strategic games, but they won't be in dynamic games.

Definition 5.1. An *extensive-form game with perfect information* is defined by the tuple $\Gamma = (\mathcal{N}, H, \mathcal{X}, \rho, u)$.

- $\mathcal{N} = \{1, \ldots, N\}$ is a finite set of N players.
- A is the set of all actions in the game.
- H is a set of sequences (finite or infinite) of *actions* in set A, called histories, which satisfies the following properties:
  - The empty sequence $\varnothing \in H$,
  - For all histories $(x^k)_{k=1}^K \in H$ (where K may be infinite) and for all $L < K$, we have $(x^k)_{k=1}^L \in H$ and $(x^k)_{k=1}^L$ is a prefix of $(x^k)_{k=1}^K$, noted $(x^k)_{k=1}^L \sqsubseteq (x^k)_{k=1}^K$,
  - For all infinite sequences $(x^k)_{k=1}^\infty$ satisfying $(x^k)_{k=1}^L \in H$ for every positive integer L, we have $(x^k)_{k=1}^\infty \in H$.

A history $(x^k)_{k=1}^{K} \in H$ is terminal if it is infinite or if there is no x^{K+1} such that $(x^k)_{k=1}^{K+1} \in H$. We define Z as the set of terminal histories.

- $\rho : H \backslash Z \to \mathcal{N}$ is the player function that assigns to each nonterminal history $h \in H \backslash Z$, the player who move after the history h.
- $\mathcal{X} : H \backslash Z \to 2^A$ is a mapping defined such as for all nonterminal history $h \in H \backslash Z$,

$$\mathcal{X}(h) := \{x \mid hx \in H\}$$

is the action set of player $\rho(h)$.
- For each player $i \in \mathcal{N}$, $u_i : Z \to \mathbb{R}$ is the player i's cost (or payoff) function.

Remark 5.1. For simplicity, the definition does not consider games where several players can play at the same time. This extension is presented in Section 5.2.3.

If H is finite, then the extensive-form game is *finite*. Otherwise, the game is called infinite. We deal with finite games. If the longest history of H is finite, then the game has a *finite horizon*. A player can have an infinite set of actions after some histories, so a game with a finite horizon can have an infinite number of terminal histories. If Z is finite, then the extensive game is finite branching.

Example 5.1 (Entry game)**.** Power generation Firm 1 (F_1) has a monopoly on the market. A second firm, Firm 2 (F_2), has the opportunity to enter the market. If Firm 2 enters, Firm 1 will have to choose how to compete: either aggressively (Fight it), or by giving up part of its market share (Adapt). An extensive game that models this situation is $\Gamma = (\mathcal{N}, H, \mathcal{X}, \rho, u)$ with:

- $\mathcal{N} = \{F_1, F_2\}$,
- $H = \{\varnothing$, Out, In, (In, Fight), (In, Adapt)$\}$,
- The terminal histories are $Z = \{$Out, (In, Fight), (In, Adapt)$\}$,
- The player function assigns the Firm 2 to the start of the game $\rho(\varnothing) = F_2$, and the firm that plays after the history "In" $\rho(\text{In}) = F_1$,
- The action set of Firm 2 available at the start of the game is $\mathcal{X}(\varnothing) = X_2 = \{Out, In\}$, and the action set of Firm 1 after the history "In" is $\mathcal{X}(\text{In}) = X_1 = \{$Fight, Adapt$\}$,
- The firms' preferences are represented by the payoff functions u_1 and u_2 such as: u_1(Out)$= 4$, u_1(In, Fight)$= -1$ and u_1(In, Adapt)$= 2$, and for the Firm 2 u_2(Out)$= 0$, u_2(In, Fight)$= -1$ and u_2(In, Adapt)$= 2$.

The basic structure representation of an extensive game is a directed *tree*. The induced tree in Example 5.1 is shown in Figure 5.1.

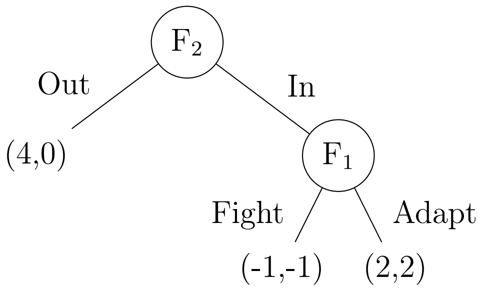


Figure 5.1.: The entry game of Example 5.1. Note that the Firm 1's payoff is the first number in the tuple.

The nodes of the tree represent game *states* that encode the full history of the play. The node at the top of the tree is the initial history $\varnothing$, which is often called the tree's root. The label in the circle indicates the player who chooses an action. In Figure 5.1, the F_2 in the circle indicates that the Firm 2 makes the first move ($\rho(\varnothing) = F_2$) of the game. The branches at a node represent the actions available to the player at the node. Each terminal node is called a *leaf* of the tree and stands for an outcome of the game. The tuple of numbers beneath each leaf provides the players' payoffs to that terminal history.

We provide another classical example with its tree representation [7, 132].

Example 5.2 (Sharing money)**.** Two people have to split two euros as follows. First, player 1 proposes a division of the sum, then player 2 can accept (A) or reject (R) the proposal. If the proposal is accepted , the money is allocated according to the proposal, otherwise both players receive nothing. The extensive game is shown in Figure 5.2.

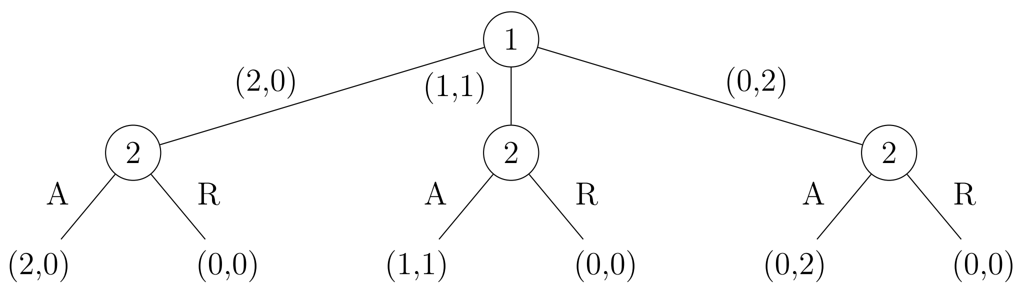


Figure 5.2.: The sharing money game of Example 5.2. Note that the player 1's payoff is the first number in the tuple.

An example where player 1 takes an action at the start of the game and then chooses again after the second player has played.

Example 5.3. Player 1 must choose between A and B. If action A is taken, player 2 must choose between C and D, otherwise between E and F. Finally, if F is selected, the first player must decide between G and H.

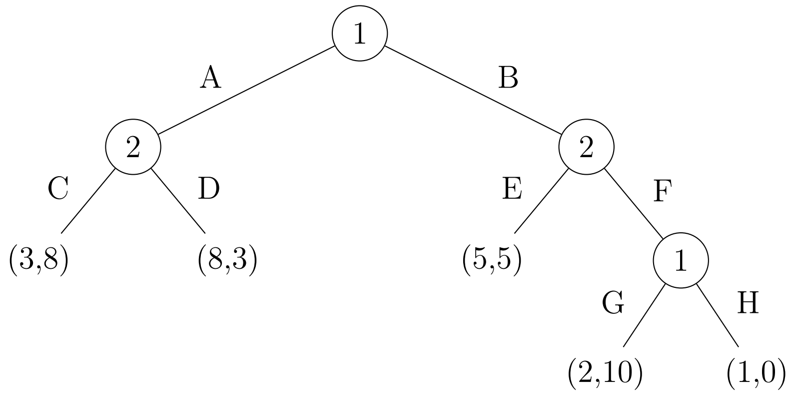


Figure 5.3.: The extensive game of Example 5.3 in which player 1 moves before and after player 2. Note that the player 1's payoff is the first number of the tuple.

For an extensive game, a player's strategy is a complete plan of action explaining how the player will play the game.

Definition 5.2. Let Γ be an extensive game with perfect information. A strategy of player $i \in \mathcal{N}$ is a function $s_i : H_i \backslash Z \to X_i$ such that $s_i(h) \in \mathcal{X}(h)$ for each $h \in H_i \backslash Z$, where H_i is the histories set for which $\rho(h) = i$.

We define $\mathcal{S}_i$ as the set of strategies available to player i, and the strategy profiles set of the game is $\mathcal{S} := \prod_{i \in \mathcal{N}} \mathcal{S}_i$. Again, we write $s = (s_1, \ldots, s_N)$ as a strategy profile and $s_{-i} \in \mathcal{S}_{-i}$ are the other players' strategies.

A strategy specifies a unique player's action for *every* history after which it is the player turn to move, even for histories that are *never reached if the strategy is followed*! We illustrate the strategy notion with the game in Figure 5.3 from Example 5.3. Player 2 chooses an action after each of the two histories A and B. In both cases there are two possible actions. A strategy of player 2 is a function that assigns either C or D to the history C, and either E or F to the history B. Then, the second player has four strategies: $\mathcal{S}_2 = \{$CE, CF, DE, DF$\}$. The first player takes a decision after the initial history $\varnothing$ and the history (B, F). The strategy function of player 1 attaches either A or B to the initial history, and either G or H to the history (B, F). Player 2 also has four strategies: $\mathcal{S}_1 = \{$AG, AH, BG, BH$\}$. Thus, the strategy indicates an action after history (B, F) even though it states that player 2 chooses A at the start of the game. The strategy specifies an action to be taken in all circumstances, even if this same strategy never reached certain states. In a sense, we can say that the AG and AH strategies are equivalent because, for any fixed choice of the other players, they lead to the same result.

Note that for the sharing money game in Fig. 5.2 from Example 5.2, player 1 has three strategies, while player 2 has eight strategies!

Given the definition of strategy for a game in extensive form, we can reuse the Nash equilibrium as a solution concept.

5.2.2. Solution concept

Nash equilibrium

As a reminder, a strategy profile is a Nash equilibrium (NE) if no player has an interest in deviating unilaterally from the strategy, given the other players' strategies (see Definition 3.12). We refer to the subsection 3.2.1 in Chapter 3 for further details.

An extensive-form game can also be converted into a normal form. Then, we can define a NE of an extensive game as a NE of the derived strategic game.

> The set of Nash equilibria of any extensive game with perfect information is the set of Nash equilibria of its strategic form.

Table 5.1-5.3 represents the associated normal-form game of the extensive-form game in Figure 5.1-5.3, respectively. Note that in all three examples, the players seek to maximize their payoffs. In an extensive game, strategies are combinations of actions, so the strategic form has exponential size [104]. An extensive-form game can be written in normal form, but the other implication is not necessarily true.

Example 5.1 (continued)**.** The extensive game is represented by the Figure 5.1. The strategy set of Firm 1 is $\mathcal{S}_1 = \{\text{Fight, Adapt}\}$ and Firm 2's strategy set is $\mathcal{S}_2 = \{\text{Out, In}\}$. Table 5.1 shows the associated normal-form game. There are two NEs: (Fight, Out) and (Adapt, In).

		Firm 2	
		Out	In
Firm 1	Fight	$(4, 0)$	$(-1, -1)$
	Adapt	$(4, 0)$	$(2, 2)$

Table 5.1.: The strategic form of the extensive game in Figure 5.1.

Example 5.2 (continued)**.** The extensive game is represented by the Figure 5.2. The strategy set of player 1 is $\mathcal{S}_1 = \{(2, 0), (1, 1), (0, 2)\}$ and Player 2's strategy set is $\mathcal{S}_2 = \{$AAA, AAR, ARA, ARR, RAA, RAR, RRA, RRR$\}$. Table 5.2 shows the associated normal-form game. There is nine NEs: ($(2, 0)$,AAA), ($(2, 0)$, AAR), ($(2, 0)$, ARA), ($(2, 0)$, ARR), ($(1, 1)$, RAA), ($(1, 1)$, RAR), ($(0, 2)$, RRA), ($(2, 0)$, RRA) and ($(2, 0)$, RRR).

		Player 2							
		AAA	AAR	ARA	ARR	RAA	RAR	RRA	RRR
	$(2, 0)$	$(2, 0)$	$(2, 0)$	$(2, 0)$	$(2, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$
Player 1	$(1, 1)$	$(1, 1)$	$(1, 1)$	$(0, 0)$	$(0, 0)$	$(1, 1)$	$(1, 1)$	$(0, 0)$	$(0, 0)$
	$(0, 2)$	$(0, 2)$	$(0, 0)$	$(0, 2)$	$(0, 0)$	$(0, 2)$	$(0, 0)$	$(0, 2)$	$(0, 0)$

Table 5.2.: The strategic form of the extensive game in Figure 5.2.

Example 5.3 (continued)**.** The extensive game is represented by the Figure 5.3. Table 5.3 shows the associated normal-form game. There is three NEs: (AG, CF), (AH, CF) and (BH, CE).

		Player 2			
		CE	CF	DE	DF
	AG	$(3, 8)$	$(3, 8)$	$(8, 3)$	$(8, 3)$
Player 1	AH	$(3, 8)$	$(3, 8)$	$(8, 3)$	$(8, 3)$
	BG	$(5, 5)$	$(2, 10)$	$(5, 5)$	$(2, 10)$
	BH	$(5, 5)$	$(1, 0)$	$(5, 5)$	$(1, 0)$

Table 5.3.: The strategic form of the extensive game in Figure 5.3.

In fact, a NE ignores the sequential structure of the extensive game. It considers the strategies as choices that are made once and for all before play begins. As a result, NEs can exhibit an undesirable characteristic in the case of extensive-form games. We consider the Nash equilibria in the Example 5.1 in order to highlight this characteristic.

The entry game has two Nash equilibria, the strategy profiles (Fight, Out) and (Adapt, In). The outcome of the game cannot be predicted precisely. We need to find a way to discriminate between these NEs focusing on the most likely outcome. On closer examination, the equilibrium (Fight, Out) appears strange. Firm 1 chooses to fight to induce the Firm 2 not to enter the market. However, if Firm 2 were to enter the market, Firm 1's payoff is -1 if it fights and 2 if it adapts. Thus, if the history "In" were to occur, Firm 1 has every interest in adapting and not carrying out its threat. Consequently, this equilibrium is based on a *non-credible threat*. Firm 1 can be confident that if it enters, then Firm 2 will choose Adapt, since Firm 2 prefers the outcome (Adapt, In) to the Nash equilibrium outcome (Fight, In). So, Firm 2 has an incentive to deviate from the equilibrium. We need to define a concept of equilibrium that takes these considerations into account.

Subgame perfect equilibrium

The concept of *subgame perfect equilibrium* (SPE) is related directly to the extensive form of a game. It aims to rule out non-credible equilibria by assuming that once a history has happened, each rational player's strategy specified an optimal action, given the other players' strategies. We need to define the notion of a *subgame* in order to provide the definition of a SPE.

Definition 5.3. Let $\Gamma = (\mathcal{N}, H, \rho, (u_i)_{i\in\mathcal{N}})$ be an extensive game with perfect information and let $h \in H$. A *subgame* of the Γ extensive game which follows the history h, is the extensive game noted $\Gamma(h) = (\mathcal{N}, H_{|h}, \rho_{|h}, (u_{i_{|h}})_{i\in\mathcal{N}})$ where

- $H_{|h}$ is the histories (sequences of actions) h' set for which $(h, h') \in H$,
- $\rho_{|h}$ is defined by $\rho_{|h}(h') = \rho(h, h')$, for each $h' \in H_{|h}$,
- $u_{i_{|h}}$ is defined by $u_{i_{|h}}(h') = u_i(h, h')$.

In an extensive game Γ, given s_i a strategy of player i and a history h, we denote by $s_{i_{|h}}$ the strategy that s_i induces in the subgame $\Gamma(h)$. For each $h' \in H_{|h}$, $s_{i_{|h}}(h') = s_i(h, h')$.

For the entry game in Example 5.1, the subgame following the history "In" is the extensive game in which Firm 2 is the only player, with two terminal histories "Fight" and "Adapt". Note that the subgame following the initial history $\varnothing$ is the entire game.

A subgame perfect equilibrium is a strategy profile s^* with the property that in no subgame can any player $i \in \mathcal{N}$ do better by choosing a strategy different from s_i^*, given the other players' strategy s_{-i}^*.

Definition 5.4 ([133])**.** Let $\Gamma = (\mathcal{N}, H, \rho, (u_i)_{i\in\mathcal{N}})$ be an extensive game with perfect information. A strategy profile s^* is a *Subgame Perfect Equilibrium* (SPE) if it induced a Nash equilibrium in each subgame of Γ (i.e., $s^*_{|h}$ is a NE of the subgame $\Gamma(h)$ for every history $h \in H$).

The notion of subgame perfect equilibrium eliminates Nash equilibria in which the players' threats are not credible. The following statement is true in general:

Every subgame perfect equilibrium is a Nash equilibrium, but NE are not necessarily SPE.

Example 5.1 (continued)**.** The NE (Fight, Out) of the entry game is not a SPE. Because, the strategy Fight is not optimal for Firm 2 since the firm is better off choosing to Adapt in the subgame following the history "In". The NE (Adapt, In) is a SPE as each firm's strategy is optimal given the other firm's strategy in the game and in the subgame following history "In".

Example 5.2 (continued)**.** Among the nine NEs in the game, only two of them: $((2,0)$, AAA) and $((1,1)$, RAA) are SPEs.

Example 5.3 (continued)**.** The strategy profile (AG, CF) is the unique SPE of the game.

We are interested in the existence results of a SPE and how to find them. The following result is known as the Kuhn's theorem.

Theorem 5.1 (Kuhn [134])**.** *Every finite extensive game with perfect information and finite horizon has a subgame perfect equilibrium.*

Note that this theorem does not claim that a finite extensive with finite horizon, has a unique SPE. Furthermore, a player may be indifferent to some outcomes.

We can find the subgame perfect equilibria by finding the Nash equilibria and checking whether each of these equilibria is subgame perfect. This task can quickly become burdensome, depending on the problem characteristics and given the exponential size of the normal form deduced from the extensive game. A common technique for identifying SPEs is to start at the end of the finite extensive game with finite horizon, and work back to the front. This is called *backward induction*. Backward induction is the process of determining a sequence of actions, in such a way that they are optimal at every decision node, for each player. It is a solution methodology applying sequential rationality. The *length* of a subgame is the length of the longest history in the subgame The procedure works as follows:

1. The backward induction considers each node that is an immediate predecessor of a terminal node (leaf), and finds the optimal actions of the rational player who moves at this node.

2. It takes these actions as given and finds the optimal actions of the players who start in the subgames of length 2.

3. At each stage k, the backward induction finds the optimal actions of the players who move first at the start of the subgames of length k, given the optimal actions found in all shorter subgames.

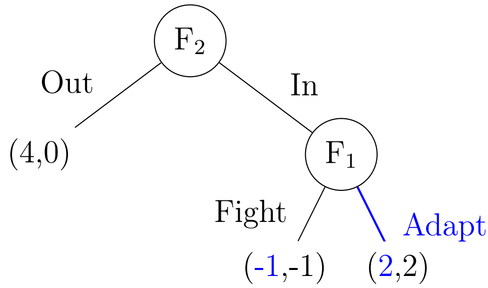
4. The process terminates after the starting point of the game is reached, the found strategies profiles are SPEs of the extensive game.

When several actions provide identical costs (or payoffs), the backward induction must traces back the implications of each optimal choice separately. Therefore, the set of strategy profiles that the algorithm of backward induction provides, is the SPE set of the extensive game.

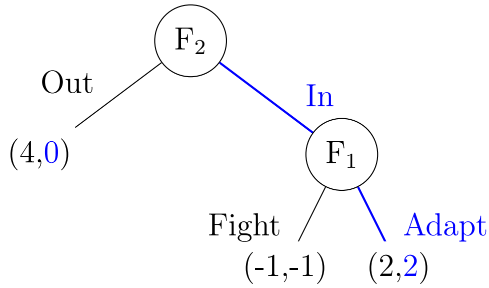
Proposition 5.1 ([132])**.** *Let $\Gamma = (\mathcal{N}, H, \rho, (u_i)_{i\in\mathcal{N}})$ be a finite horizon extensive game with perfect information. The set of subgame perfect equilibria of the game Γ is equal to the set of strategy profiles isolated by the algorithm of backward induction.*

We apply backward induction to one of the examples described in this section.

Example 5.1 (continued)**.** We take the entry game, which involves two power generation firms that maximize their payoff functions. It has been shown that the game has only one SPE, the strategy profile (Fight, In) with payoffs $(2, 2)$. According to Proposition 5.1, the backward induction should result in the same strategy profile. Figure 5.4 shows the backward induction process on the entry game.



(a) Step 1



(b) Step 2

Figure 5.4.: Application of backward induction on the entry game of Example 5.1 with (a) the first step of the procedure and (b) the second and final step.

We start by considering the actions of Firm 1 in the subgame following "In". The action Adapt is an optimal choice for Firm 1 as $2 \geqslant -1$ (see Figure 5.4a)). The entire game is the only subgame of length 2, at which Firm 2 moves. Given the optimal actions in the subgame following "In", Firm 2's best response is to enter as $2 \geqslant 0$ (see Figure 5.4b)). In fact, we can say that Firm 2 anticipates the optimal action of Firm 1 and chooses In at the start of the game. Then, the backward induction procedure proposes the equilibrium (Adapt, In), which is what was expected.

Remark 5.2. The presence of the same costs (or payoffs) introduces a degree of strategic flexibility for players, because some choices may have not impacted the final outcome. This flexibility can be used to model situations where external elements or not modeled criteria influence the decision (e.g., social, economic or environmental preferences). In this way, players can decide between actions using secondary criteria. For example, a player might choose an action based on the *lexicographical order*.

Definition 5.5. Let $(X_i, \leqslant_i)$, $i = 1, \dots, k$ be partial orders. The lexicographical order $\preceq$ on $X_1 \times \dots \times X_k$ is defined as: $(x_1, \dots, x_k) \preceq (y_1, \dots, y_k)$ if either $(x_1, \dots x_k) = (y_1, \dots, y_k)$, or there exist a $1 \leqslant d \leqslant k$ such as $x_d \leqslant_d y_d$, $x_d \neq y_d$ and for all $i = 1, \dots, d-1$, $x_i = y_i$.

So, $(x_1, \dots, x_k) \preceq (y_1, \dots y_k)$ if either the two k-uplet are equal, or in the first coordinate d from where they differ $x_d \leqslant_d y_d$. As a reminder, $\leqslant_X$ is a partial order on X if it is a transitive, reflexive and antisymmetric binary relation on X.

Example 5.4. The lexicographic order on $\{0,1\} \times \{0,1\}$ usually ordered, gives $(0,0) \preceq (0,1) \preceq (1,0) \preceq (1,1)$.

Note that the lexicographic order is useful, but it loses some SPEs in the process.

5.2.3. Simultaneous moves extension

The Definition 5.1 of an extensive game with perfect information assumes that after each sequence of events, only one player chooses an action with knowledge of each player's previous actions. In the next part of this thesis, we investigate situations where players simultaneously choose their actions after some histories. Each player knows the previous actions of every player, but not those they are currently taking at the same time. We extend the definition as follows.

Definition 5.6 ([7])**.** An *extensive game with perfect information and simultaneous moves* is defined by the tuple $\Gamma = (\mathcal{N}, H, \rho, (u_i)_{i\in\mathcal{N}})$ where,

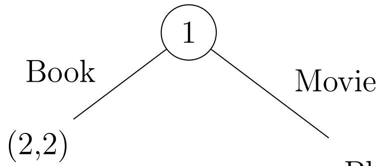
- $\mathcal{N}$, H, Z and $(u_i)_{i\in\mathcal{N}}$ are the same as in Definition 5.1.
- ρ is a function that assigns a set of players to each nonterminal history $h \in H \backslash Z$.
- H and ρ jointly satisfy the condition that for every nonterminal history $h \in H\backslash Z$ there is a collection $\{\mathcal{X}_i(h)\}_{i\in\rho(h)}$ of sets for which

$$\mathcal{X}(h) = \{x \mid hx \in H\} = \prod_{i\in\rho(h)} \mathcal{X}_i(h),$$

  such as for all $i \in \rho(h)$, $\mathcal{X}_i(h)$ is the set of actions available for player i after the history h.

In an extensive game with perfect information and simultaneous moves, a history is a sequence of vectors. For each vector x^k, the components are the actions chosen by the players having to make a decision after the history $(x^l)_{l=1}^{k-1}$. We use the following example to illustrate this type of game.

Example 5.5 (Variant of Battle of Sexes [132])**.** The first player must choose between staying home to read a book or going to the cinema. If he opts to read, the game ends. If he decides to see a movie then, as in Example 3.3, he and the second player then select, independently and without knowing the other's choice, a film from two options: A or B. Both players would rather see their favorite film together than have the first player reading at home. However, the latter outcome is preferable to watching a film they do not like together. The worst-case scenario for both players is that they choose different films. In this context, both players maximize their payoffs. The extensive game with perfect information and simultaneous moves is shown in Figure 5.5, with



		Player 2 A	Player 2 B
Player 1	A	$(3, 1)$	$(0, 0)$
	B	$(0, 0)$	$(1, 3)$

Figure 5.5.: The variant of battle of sexes of Example 5.5.

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- $\mathcal{N} = \{1, 2\}$,
- $Z = \{$Book, (Movie, (A, A)), (Movie, (A, B)) (Movie, (B, A)), (Movie, (B, B))$\}$,
- $\rho(\varnothing) = 1$ and ρ(Movie)= $\{1, 2\}$,
- The player 1's action set available at the start of the game is $\mathcal{X}_1(\varnothing) = \{$Book, Movie$\}$ and her set of actions after the history "Movie" is $\mathcal{X}_1$(Movie)= $\{$A, B$\}$. Player 2's action set after the history "Movie" is $\mathcal{X}_2$(Movie)= $\{$A, B$\}$.

The definition of a strategy for a player $i \in \mathcal{N}$ is identical to that in Definition 5.2, except that "$\rho(h) = i$" is replaced by $i \in \rho(h)$. Furthermore, the concept of strategic form for an extensive game with perfect information and simultaneous moves is the same as before, and a strategy profile is a Nash equilibrium of the extensive game if and only if it is a NE of its strategic form.

Example 5.5 (continued)**.** The extensive game with simultaneous moves is represented by the Figure 5.5. The strategy set of player 1 is $\mathcal{S}_1 = \{$(Book, A), (Book, B), (Movie, A), (Movie, B)$\}$ and player 2's strategy set is $\mathcal{S}_2 = \{$A, B$\}$. Table 5.4 shows the associated strategic form game. There is three NEs: ((Movie, A), A), ((Book, A), B) and ((Movie, B), B).

Player 1		Player 2: A	Player 2: B
	(Book,A)	$(2,2)$	$(2,2)$
	(Book,B)	$(2,2)$	$(2,2)$
	(Movie,A)	$(3,1)$	$(0,0)$
	(Movie,B)	$(0,0)$	$(1,3)$

Table 5.4.: Strategic form of the game in Example 5.5.

In addition, a subgame perfect equilibrium of an extensive game with perfect information and simultaneous moves is defined in the same way as previously established in Definition 5.4, with the exception that "$i \in \rho(h)$" instead of "$\rho(h) = i$". Nevertheless, the Kuhn's theorem 5.1 ensuring the existence of at least one SPE, cannot be extended to the case of finite extensive games with perfect information and simultaneous moves. In fact, a strategic game can be seen as an extensive game with simultaneous moves of length 1. Moreover, a finite normal-form game may not have a NE (e.g., the Matching Pennies game in Example 3.5 on page 57), which explains why the Kuhn's theorem does not extend in this case.

Remark 5.3. Although it is not developed in this thesis, it should be noted that the Kuhn's theorem for mixed strategies (where each player chooses a probabilistic distribution over their possible actions) applies to extensive-form games with simultaneous moves. The reader can refer to the sources [7, 132] for more information.

As before, we can use backward induction to obtain all the SPEs of an extensive game with perfect information and simultaneous moves that has a finite horizon. The only complexity arises from the fact that some (or perhaps all) of the situations we need to analyze involve multiple players choosing actions simultaneously. When this happens, we must identify a list of actions for the players who move at the beginning of each subgame, ensuring that each player's action is optimal, taking into account the simultaneous choices of others and their behavior throughout the rest of the game [132]. Which is similar to the reasoning used to determine the Nash equilibria of a strategic game.

Example 5.5 (continued)**.** We apply backward induction to the extensive game in Figure 5.5 which have finite horizon. The process goes as follows. We consider the subgame following the history "Movie", there are two NEs: (A, A) et (B, B). The only subgame of length 2 is the entire game, at which the first player moves. If the outcome of the subgame that follows "Movie" is (A, A) then player 1's best response is to see a movie as $2 \leqslant 3$. If the outcome of the subgame following "Movie" is (B, B), then player 1's optimal choice is to read a book at home as $2 \geqslant 1$. Therefore, the backward induction implies that the game has two SPEs: ((Book, B), B) and ((Movie, A), A).

Remark 5.4. The results and reasoning presented remain valid in a context expanded to include generalized Nash equilibria (GNEs).

5.2.4. Exogenous uncertainty extension

We are interested in situations involving exogenous uncertainties. Uncertainty is said to be exogenous if it does not depend on the decisions of the system's agents. Otherwise, the uncertainty is endogenous. We extend the model of an extensive-form game with perfect information given in Definition 5.1 to cover problems with some exogenous uncertainty. We also suppose that the extensive game is finite.

Definition 5.7 ([7])**.** An *extensive game with perfect information and chance moves* is defined by the tuple $\Gamma_c = (\mathcal{N}_c, H, \rho, f_c, (u_i)_{i\in\mathcal{N}})$ where,

- $\mathcal{N}_c = \mathcal{N} \cup \{c\}$ is a finite set with $\mathcal{N} = \{1, \ldots, N\}$ a finite set of N players and c represents the *nature* or *chance*.
- H and Z are the same as in Definition 5.1,
- $\rho : H\backslash Z \to \mathcal{N}_c$ is a function such as if $\rho(h) = c$, then chance determines the action taken after the history $h \in H$,
- For each $h \in H$ with $\rho(h) = c$, $f_c(\cdot|h)$ is a probability measure on $\mathcal{X}(h)$ defined as $f_c(x|h)$ is the probability that $x \in \mathcal{X}(h)$ occurs after the history $h \in H$,
- For each player $i \in \mathcal{N}$, $u_i : Z \to \mathbb{R}$ is the utility function of player i.
- The player's preferences are defined over the set of lotteries (or probability distributions) over the terminal histories.

Note that each probability measure is assumed to be independent of every other such measure. Despite the inclusion of chance moves, the player who makes a decision after nature knows the previous action made by the other players before him. So, although there is uncertainty, we still call this a game with perfect information. We say that the game with perfect information is conditional on the realization of uncertainty.

Example 5.6 ([132])**.** A first player chooses A or B. If he takes A, the game ends with payoffs $(1, 1)$. Otherwise, the game ends with probability 0.5 and payoffs $(3, 0)$, while with probability 0.5 the second player takes a decision between C and D, which yield payoffs $(0, 1)$ and $(1, 0)$ respectively. The game is represented in Figure, where c denotes chance and the number beside each chance action is the associated probability.

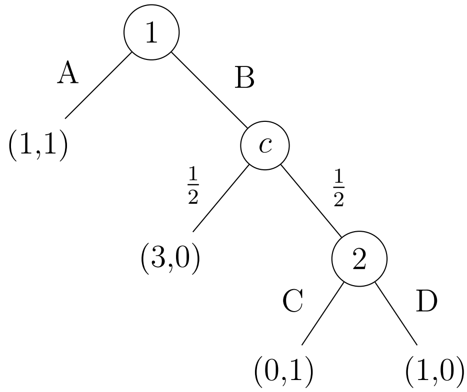

Figure 5.6.: The extensive game with chance moves of Example 5.6. Note that the player 1's payoff is the first number of the tuple and label c denotes the chance.

The Definition 5.7 does not affect the notion of strategy (Definition 5.2). However, because of the random nature of some moves, the outcome of a strategy profile is a probability distribution over the terminal histories. In such a framework, players do not directly minimize the costs associated with terminal histories, as these costs are uncertain due to exogenous uncertainty. Instead, rational players make decisions under uncertainty that minimize their expected utilities, according to the Expected Utility Theory (EUT). More information about this theory and the concept of the expected utility are provided in Section 5.3.1.

The definition of a subgame perfect equilibrium for an extensive game with chance moves, is defined in the same way as established in Definition 5.4. As before, we can use backward induction to determine the SPEs set of the game; we have the following result.

Theorem 5.1 holds for an extensive game with perfect information and chance moves.

Example 5.6 (continued)**.** We consider the extensive game with chance moves in Fig. 5.6. Player 1's strategy set is $\mathcal{S}_1 = \{A, B\}$ and player 2's strategy set of $\mathcal{S}_2 = \{C, D\}$. We use backward induction to obtain the SPEs. In the subgame where player 2 is the first to move, this player's best response is C. We consider the consequences of player 1's decision. If he chooses A, then he gets the payoff 1. If he chooses B, then he obtains 3 with probability 0.5 and 0 with probability 0.5, so the expected payoff is:

$$U_1 = \frac{1}{2}.3 + \frac{1}{2}.0 = \frac{3}{2}.$$

Then, player 1's best response is B as $1 \leqslant 3/2$. As a result, the unique SPE is when player 1 takes B and player 2 chooses C.

5.3. Prospect theory

We assume that the reader is familiar with the concepts of probability theory [135].

In many situations, individuals are faced with choices involving several options with uncertain outcomes. When these outcomes are associated with well-known or estimable probabilities, the process is referred to as decision-making under risk. These options are often called prospects or lotteries. Each possible outcome leads to consequences that differ in value or preference for the decision-maker, influencing her choices.

Definition 5.8. A prospect (or lottery) L is a probability distribution over a set of outcomes $X = \{x_1, \ldots, x_n\} \subseteq \mathbb{R}$. The probability of each outcome occurring is p_i, for all $i = 1, \ldots n$. The prospect is represented as

$$L = (x_1, p_1; x_2, p_2; \ldots ; x_n, p_n), \quad \sum_{i=1}^{n} p_i = 1. \tag{5.1}$$

The set of prospects is noted $\mathcal{L}$.

There are two approaches to decision analysis models:

1. Normative decision models. These models determine optimal decisions that a rational individual should take. A rational agent seeks the actions that will be most profitable for her, and to do this he has constant access to relevant information and knowledge, as well as unlimited processing capacity. The agent is aware of the available options and has clear preferences.

2. Descriptive decision models. These models aims to describe and predict how individuals actually make decisions. In this context, an agent possesses bounded rationality [136, 78]. Bounded rationality refers to

the idea that individuals, although intentionally rational, make decisions based on their limited access to information, their restricted cognitive capacities and some heuristics. This leads them to outcomes that are satisfactory, but not always optimal.

One of the most important normative models for analyzing decisions under risk is the *expected utility theory.*

5.3.1. Expected utility theory

The Expected Utility Theory (EUT) is a normative decision model established by Von Neumann and Morgenstern (VNM) in their founding book *Theory of Games and Economic Behavior*, in 1944 [8]. In addition, this theory also constitutes a fundamental foundation for the game theory. The EUT is based on a series of axioms describing the behavior of a rational individual who must make choices in a risky situation. These axioms are completeness, transitivity, continuity, independence of irrelevant alternatives and monotonicity.

In the EUT and economics framework, individuals do not directly compare the possible outcome of their choices, but rather the utility associated with each outcome. The utility is a measure of well-being or satisfaction obtained by acquiring a certain number of commodities or services. Thus, it is assumed that a decision-maker possesses a utility function $u : \mathbb{R} \to \mathbb{R}$ that represents her preferences over a set of possible outcomes. Therefore, the EUT assumes that when faced with probabilistic outcomes, a rational agent chooses the action that maximizes (or minimizes) the expected value of her utility function $u : \mathbb{R} \to \mathbb{R}$. For a discrete case, the expected utility of a prospect L is defined as

$$U(L) = \mathbb{E}(u(L)) := \sum_{i=1}^{n} p_i u(x_i), \tag{5.2}$$

where $u(x_i)$ is the utility associated to the outcome x_i. Hence, an individual is said to prefer the prospect L to L' if and only if $U(L) \geqslant U(L')$.

It is worth highlighting that in EUT, decisions are made in terms of final states of wealth, i.e., the individual's overall situation after considering the outcomes of a decision. Furthermore, the curvature of a utility function reflects the individual's attitude to risk. A concave utility function translates a risk aversion, while a convex function indicates a preference for risk. If the function is linear, the decision-maker is risk-neutral.

5.3.2. Prospect theory

Although expected utility theory (EUT) is a powerful normative model for analyzing rational choices in risk situations, numerous experimental results have revealed anomalies in its application to real-life decision-making.

In practice, empirical studies ([137, 77, 80]) have shown that, in uncertain and risky situations, human players may not act in accordance with the rational behavior established by expected utility theory, and so game theory. In fact, people are irrational, and EUT is inadequate to describe the actual behavior of individuals in decision-making under risk. An emblematic example is the Allais paradox [137], which shows that individuals' choice can breach the axiom of independence in EUT. The following example presents a variant of this paradox.

Example 5.7 ([77])**.** A questionnaire is presented to different participants. For each problem, the agents must choose one of the proposed prospects. The survey is given as follows:

- Problem 1:

  A: An 80% chance of getting a payoff of 4000€,

  B: A 100% chance of getting a payoff of 3000€.

- Problem 2:

  C: A 20% chance of getting a payoff of 4000€,

  D: A 25% chance of getting a payoff of 3000€.

The results obtained for the first problem show that 80% of agents take prospect B. Under EUT, these preferences imply that the expected utility of prospect B is strictly higher than that of A (i.e., $U(\text{A}) < U(\text{B})$). We note that C and D can be obtained from A and B, we have that C=(A, 0.25) and D=(B, 0.25). So, according to EUT, if B is preferred to A, then D should be preferred to C. In reality, 65% of participants prefer C to D. The individuals have therefore not respected the expected utility theory.

In order to describe non-rational human behavior in decision-making under risk, the EUT has been set aside in favor of modern theories such as the Prospect Theory (PT) proposed by Kahneman and Tversky.

The research of Kahneman and Tversky has left a major impact on the study of bounded rationality, enabling a more nuanced understanding of human decision-making and judgments in situations of uncertainty. Far from always behaving rationally and consistently, individuals are influenced by personal preferences and their restricted access to information and cognitive capacities. In this way, Kahneman and Tversky explored heuristics that individuals use to simplify complex decisions as well as the cognitive biases involved [138, 139]. These psychological biases manifest themselves in a variety of judgment tasks, including predictions of future events or evaluations of available evidence [78]. Their work revealed that these mental shortcuts, although effective in many cases, lead to systematic errors in regard to the optimal beliefs and choices obtained from rational-based models.

In addition to the cognitive biases studies, Kahneman and Tversky developed a model to describe boundedly rational decision-making under risk, based on their experimental results. Prospect Theory (PT) is a descriptive decision model, which is one that seeks to describe how individuals actually make their decisions, as opposed to a normative model such as EUT, which looks for the best choice to be made for an individual. The original formulation in [77] was refined as *Cumulative Prospect Theory* (CPT) in [81] and then as *Smooth Prospect Theory* (SPT) in [140] for continuous distributions. The fundamental characteristics of prospect theory remain faithful to the original version even though its mathematical formulation has evolved. The term prospect theory (PT) is used throughout this thesis.

Prospect theory is based on the principle that non-rational individuals do not maximize objective expected utility but rather a global subjective value, according to their perception of outcomes and associated probabilities. This global value V is characterized by two fundamental components:

1. A subjective value function v, which reflects how an individual evaluates outcomes in relation to a reference point;
2. A probability weighting function w, which translates the biased perception of probabilities by an individual, influencing their decision-making.

These two concepts capture the behaviors observed in real decision-making, and are described in the following sections.

5.3.3. Subjective value function

Value carriers are seen as changes in wealth or well-being rather than absolute payoffs. In PT, these changes are perceived as gains or losses regarding a *reference point* noted r. This principle is also known as the framing effect. Each individual has her own reference point, depending on her perception of the problem, objectives, knowledge and other personal choice heuristics. For example, a gain of 50€ will be perceived differently by a high-income person as opposed to a low-income individual. Besides, the exact interpretation of the reference point varies in the literature. In the founding paper of PT [77], Kahneman and Tversky define the reference point as the individual's initial situation (current wealth or well-being). It is then fixed and normalized at zero. In recent papers [141, 13, 142], the reference point represents an agent's expectation of the problem, and is no longer normalized to zero. The subjective value function is then defined on results representing the individual's payment and describes how agents evaluate gains and losses. This outcome is said to be a gain if it is at least as good as the reference point, otherwise it is said to be a loss.

The value function should be concave for gains ($v'' < 0$) and convex for losses ($v'' > 0$), reflecting the diminishing sensitivity. In both gain and loss regions, an individual's sensitivity to a marginal change in her subjective value decreases with distance from the reference point. For instance, a change from 0 to 100€ is perceived as more significant than going from 1000€ to 1100€. Furthermore, the value function is generally steeper for losses than for gains. This captures the phenomenon that an individual is more sensitive to losses than to gains of the same amount, i.e., humans manifest loss aversion. In other words, losing 100€ is perceived as more painful than gaining 100€ is satisfying. Mathematically, the subjective value function $v : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is represented in [81] by equation (5.3) and displayed in Figure 5.7.

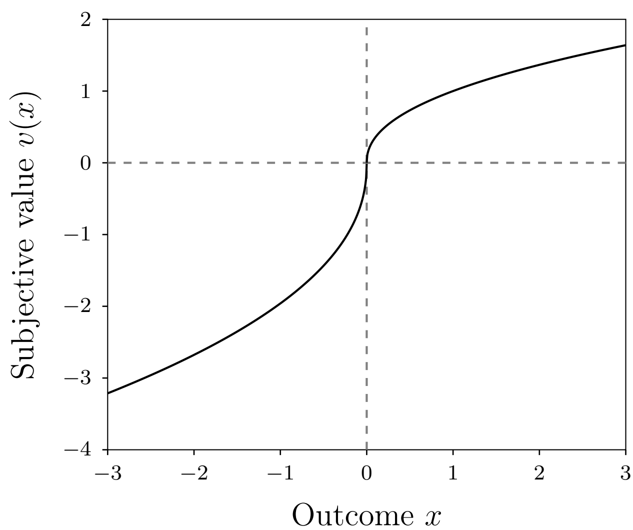

Figure 5.7.: Subjective value function v as defined in (5.3). For this illustration, $r = 0$ and the parameters are $\alpha = \beta = 0.45$ and $\lambda = 1.96$.

We have

$$v(x_i, r) := \begin{cases} (x_i - r)^\alpha & \text{if } x_i \geqslant r \\ -\lambda(r - x_i)^\beta & \text{if } x_i < r, \end{cases} \tag{5.3}$$

where x_i is an outcome, r is the reference point, α and $\beta \in]0, 1[$ represent the diminishing sensitivity speed of gains and losses respectively, and $\lambda > 1$ the loss aversion coefficient. Kahneman and Tversky estimated the value parameters of the subjective function at $\alpha = \beta = 0.88$ and $\lambda = 2.25$, on the basis of experiments carried out with real people [81]. Recent empirical studies on companies in different sectors, provide $\alpha = \beta = 0.45$ and $\lambda = 1.96$ [143]. Further parameters estimates by country can be found in [144].

Remark 5.5. The reference point selection is a major operation of this theory, but this selection is usually dependent on the context in which the theory is applied. Furthermore, in a specific context, it can be difficult for an analyst to determine people's reference points and to translate certain choice problems into terms of gains and losses [145]. In the framework of game theory, a player's reference point could depend on internal factors such as the opponents' payoffs [146]. A PT extension that allows reference points to be uncertain is developed in [147]. Recently, the idea of a dynamic reference point is increasingly being considered in practical cases [148].

5.3.4. Probability weighting function

In PT, individuals distort the probabilities of uncertain outcomes in real decision-making, in contrast to the EUT hypothesis where agents are regarded as statisticians who judge probabilities objectively. The probability weighting function w assigns a value called the decision weight, which represents the subjective probability of an objective probability.

Remark 5.6. For all $i = 1, \ldots, n$, the decision weight $w(p_i)$ is not a probability. Indeed, the function w does not respect the additivity axiom of probability theory. Then, the sum of weights for a prospect may not be equal to 1. In fact, this is a type of measure called capacity defined by Choquet [149].

Individuals tend to overweight low probabilities and underweight medium and high probabilities. Diminishing sensitivity is an integral part of the subjective value function, but also manifests in the probability weighting function. As a reminder, this psychological principle asserts that humans are less sensitive to variations in probability as it gets further away from 0 and 1 for both gains and losses. For example, a change of 0.1 has more impact when it shifts an initial probability from 0.9 to 1 or from 0 to 0.1 than when it modifies 0.6 to 0.7 [81]. In addition, decision-makers often display a preference for certain over uncertain gains and uncertain over certain losses. The w function in prospect theory captures four fundamental attitudes of individuals towards risk observed in the experimental results, summarized in the table 5.5.

	Gains	Losses
Low probabilities (possibility effect)	Risk seeking	Risk aversion (provided losses are not extreme)
Medium and large probabilities	Risk aversion	Risk seeking

Table 5.5.: Risk attitudes.

The original mathematical formulation of $w : [0,1] \to [0,1]$ proposed in [81], is given by

$$w(p_i) := \frac{p_i^\gamma}{(p_i^\gamma + (1-p_i)^\gamma)^{\frac{1}{\gamma}}} \tag{5.4}$$

where p_i is the probability associated to the outcome x_i and γ is the probability weighting parameter. Based on experimental results at the individual level, it is estimated that $\gamma = 0.65$. There is as yet no clear real value at company level [150]. Other parameters estimates can be seen in [144]. As shown in Figure 5.8, the probability weighting function has an "inverted S" curve with several properties:

1. $w(0) = 0$ and $w(1) = 1$,
2. It is asymmetrically reflected at a point $\tilde{p} \in]0,1[$ such as $w(\tilde{p}) = \tilde{p}$,
3. For all $p \in]0, \tilde{p}[$, the function is concave and $w(p) > p$,
4. For all $p \in]\tilde{p}, 1[$, the function is convex and $w(p) < p$.

5.3.5. Global value and conclusion

The global value V of a prospect L is calculated by combining the subjective value function v (Section 5.3.3) and the probability weighting function w (Section 5.3.4). This combination provides the main equation of PT defined as

$$V(L) := \sum_{i=1}^{n} w(p_i) v(x_i, r). \tag{5.5}$$

The decision-maker will therefore select the prospect that maximizes the overall value V.

Prospect theory is a promising framework for modeling the non-rational prefer-

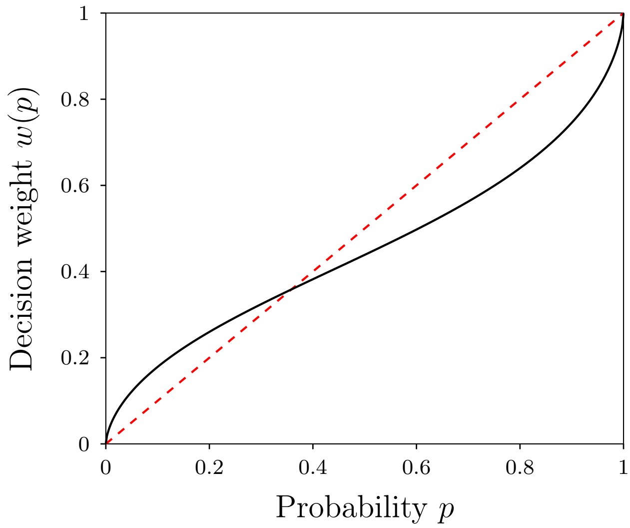

Figure 5.8.: A probability weighting function w as defined in 5.4. For this illustration, $\gamma = 0.65$.

ences of humans in real-life decision-making under risk. Unlike EUT, which assumes objective probability and rational utility, PT incorporates subjective evaluation of both probabilities and outcomes, reflecting observed deviations from rational behavior. In other words, PT captures the nuances of experimentally observed human behavior such as loss aversion, diminishing sensitivity and probability distortion. Hence, prospect theory is a milestone in behavioral economics and decision theory. Prospect theory inspired various extensions and theoretical development, for interested readers we refer to [151] for an overview.

Although PT was originally developed to analyze choices involving monetary prospects, its application has since been extended to many fields. In recent years, it has been employed in a variety of sectors, including different aspects of the energy sector [141, 152, 80, 153, 86, 13, 154, 155, 150], communication and cybersecurity [156], health, etc. In Chapter 6, we use prospect theory in the renewable energy communities framework, to model the non-rational decision-making of end-users with heterogeneous preferences.

CHAPTER 6.

New Member Integration Decisions in Renewable Energy Communities under Prospect Theory

Although renewable energy communities have attracted growing interest in recent years as an innovative contributor to the energy transition and to a prosumer-centric power system, their implementation has yet to overcome considerable challenges (Section 2.5.3 in Chapter 1). This chapter explores a crucial aspect that is still not widely studied in the literature: the integration of new members into an existing renewable energy community. The topic is especially relevant to the viability of energy communities, which are called upon to expand by integrating members with various characteristics and objectives. Against this background, the preceding chapters have laid the foundations for the analysis proposed in this chapter. Chapter 4 focused on the day-ahead energy resources scheduling problem, highlighting the operational aspects of managing energy resources and exchanges within communities. Chapter 5 introduced the theoretical concepts of extensive games and prospect theory, offering powerful analytical tools for modeling sequential strategic behavior and decision-making under risk.

The content of this chapter is based on the following publication:

- [27] L. Sadoine, Z. De Grève and T. Brihaye, "New Member Integration Problem in Renewable Energy Communities: An Extensive Game Study with Prospect Theory," in preparation.

6.1. Introduction

6.1.1. Context

European directives [6, 19] state that participation in an energy community must be open and voluntary, based on transparent and non-discriminatory criteria. Similarly, any member wishing to leave the community has the right to a fair and non-discriminatory exit procedure. However, there are no further details concerning these procedures. The absence of common standards leaves a number of gray areas, which can give rise to uncertainties and concerns for energy communities. More precisely, the impact of a user's integration or exit on community dynamics is not fully anticipated in the literature. Indeed, entry and exit processes raise specific issues for the stability and efficiency of the community: the arrival of a new user or the departure of a member can affect energy flows, costs, self-consumption and self-sufficiency rates, etc. In addition, the diversity of user profiles (in terms of consumption, production or flexibility) may require adjustments to strategies and recommendations for energy exchanges and consumption within the community. To the best of our knowledge, the analysis of the dynamics of community members after a change in their composition (entry or exit), remains relatively unexplored in the literature. This lack of scientific references and regulations is a barrier to the expansion of energy communities.

6.1.2. Related work

The authors in [20] propose a methodology to help an existing energy community to select new members or guide investment decisions with the aim of maximizing shared benefits within the community. Two heuristic metrics are introduced to assess whether the community needs more generation or consumption, and to estimate the suitability of each candidate. The assessment of more battery capacity is also performed heuristically. Simulations on a case study in France show that the ranking of candidates by these metrics aligns closely with the results of a more precise optimization method, but with faster computation and better explainability. The paper [69] develops an energy community management model incorporating a fair revenue-sharing system and exit clauses to find the optimal sizing of communities and to enhance cooperation. The exit clauses require users leaving the community to pay compensation that decreases over time, to mitigate the impact of their departure. These costs are calculated to ensure that the other members do not suffer financial losses, thus guaranteeing economic stability for the energy community. In practice, exit costs are significantly reduced after ten years, enabling users to leave without significant financial impact after this period. In [70], Perger and Auer focus on the dynamic participation of prosumers in an energy community with peer-to-peer trading by allowing adjustments over time, including users' entries and exits. It uses a bi-level optimization model where the upper-level problem minimizes an objective function that includes the prosumers' cost-saving and emission-saving preferences to determine the optimal profile of a new member, whereas the lower-level problem maximizes the collective welfare of the community according to a willingness-to-pay criteria. Once departure and new commitment decisions have been made, the optimal community configuration is recalculated for each period, ensuring that members are well aligned with collective energy and economic objectives.

This chapter proposes an original approach to the New Member Integration Problem (NMIP) into an existing renewable energy community, in which the interactions between initial members of a REC and the potential newcomer are considered. In addition to the initial energy profile, we consider that an external user has the possibility of investing in additional means of production or energy storage. However, the main objective of this chapter is not to assess the impact of a specific investment model, but rather to evaluate the overall effect of a new member, with or without additional investments, on the existing community. We refer to other works for a more in-depth exploration of the dynamics specific to investment models in energy communities, such as [157, 158, 159]. Therefore, this chapter focuses on the NMIP, perceived from a long-term (LT) planning horizon, while considering their repercussions on short-term (ST) management. We adopt an approach based on Game Theory, and more specifically extensive-form games, to model the dynamics of the problem's decision-making process. Note that extensive games are not new to smart grids and the power systems literature, and have been widely applied through Stackelberg game models for instance [160, 12, 14, 13, 161, 84]. However, their use for the NMIP in a renewable energy community with demand-side management schemes (as in design D2 of Chapter 4 for instance), remains innovative. Our approach takes into account the various stakeholders and their specific objectives, incorporating their preferences and interactions. In addition, extensive games capture the sequential structure of the NMIP decision-making process, i.e., are able to quantify the impact of the order of players' decisions on the model outcomes. The solutions of these games, known as Subgame Perfect Equilibrium (SPE), provide a through and robust plan of action, and they eliminate non-credible threats by ensuring that only coherent, rational decisions are made (see Chapter 5).

Standard modeling assumptions in energy communities consider that prosumers aim to minimize their total costs. Nevertheless, end-users' growing awareness of current energy and ecological priorities, as well as the potential economic, social and environmental benefits that a community can provide, have made it relevant to examine the results of SPEs when stakeholders show more various and heterogeneous preferences. In this context, we study the SPEs obtained when candidates and the REC adopt different criteria for their long-term goals. For reasons of simplicity and readability, we limit our analysis to five main criteria, but encourage readers to explore other relevant criteria, such as those proposed in [67, 62, 162].

Currently, most of the literature considers decision-makers as perfectly rational agents and relies more generally on the expected utility theory (EUT) [8] (Section 5.3.1). In this way, risk measures such as expected shortfall or conditional value-at-risk (CVaR) are commonly used in the energy sector [163, 64, 164, 17]. Empirical studies have shown that an agent's subjective perception and other cognitive biases regarding opponents, results and uncertainty can play a decisive role in the agent's decisions and, consequently, in final outcomes [80]. However, as a normative model, the EUT fails to capture these elements and to predict an individual's real decision-making when facing uncertainty. Kahneman and Tversky introduced prospect theory (PT) as an alternative to describe the decision-making of bounded rational individuals under risk [77],[81]. This Nobel-prize-winning theory has already been applied in the context of power systems. For instance, energy management combined with DSM strategies has been the subject of numerous studies [152, 86, 165]. Wang et al. [152] explores the role of subjective perceptions of end-users in smart grid DSM programs, comparing decisions based on EUT and PT. They show that taking non-rational behavior into account can significantly influence DSM participation rates and performance. In [86], Etesami et al. extend the analysis by introducing stochastic games under uncertainty, incorporating PT to model prosumer perceptions. Unlike more static approaches such as in [152], this paper proposes multi-period dynamics and a distributed algorithm, showing how subjective behaviors influence global energy management decisions over multiple time horizons. Good [165] integrates behavioral economic theory into the modeling of energy demand response, enabling the design of more effective energy policies, taking into account cognitive biases and individual preferences. Energy trading between participants of a microgrid or an energy community, and in smart grids has also been widely studied. El Rahi et al. [141] model a noncooperative game between prosumers, showing that the framing effect and probability weighting under PT reduce the volumes of energy exchanged compared to classical game theory. While in [13], the authors introduce a

Stackelberg game between an energy supplier and prosumers, analyzing the impact of future price uncertainty and subjective perceptions on company profit and network load. Dorahaki et al. [155] present an energy community with a centralized peer-to-peer (P2P) energy trading framework based on a modified version of PT, incorporating time discount effect to model end-users behavior under risk. It highlights the impact of subjective perceptions on trading decisions and on the overall performance of energy communities, while Andriopoulos et al. [166] propose a local energy market architecture based on a cooperative game and a pricing algorithm inspired by PT. By combining a method of profit allocation via Shapley value with the representation of non-rational prosumers' behavior, this approach promotes fairness, local self-sufficiency and better integration of renewable energies into energy communities. Investment decisions in energy assets were discussed from different angles with the PT framework, although the literature is less abundant in that respect. At the individual level, [153] studies the factors influencing households' choices to invest in PV panels, integrating behavioral and economic elements. At the organizational level, Tao et al. [150] apply PT to model power plant investment decisions, showing the impact of bounded rational behavior on the LT choices of generation companies. A sensitivity analysis considers the influence of parameters such as reference point dependence and loss aversion, revealing that these factors slow down the adoption of renewable energies. These works underline the importance of taking into account both collective dynamics and individual motivations for ST and LT decisions.

6.1.3. Contributions

The present chapter builds on these foundations to integrate heterogeneous preference criteria, model behavior under risk via prospect theory, and analyze the integration of a new member with possible investment into an existing energy community. The aim is not only to enrich the research, but also to provide practical elements for the development of more robust rules and models for future energy communities. The main contributions presented in this chapter are summarized as follows.

1. We present an original approach of the new member integration problem into an existing REC, modeled using extensive games. The problem considers both long-term strategic decisions (investments, internal price adjustments) and short-term decisions (day-ahead schedules). Our theoretical framework is general and offers enough flexibility to encompass a variety of scenarios and stakeholder preference criteria (economic and environmental). In addition, prospect theory is used to model the bounded

rationality of participants, more specifically on their perception of retail import prices, providing a better understanding of their behavior under uncertainty and risk.

2. We apply our NMIP models to a detailed case study:

   a) We first compare our model outcomes with the results of the heuristic methods proposed in [20]. We show that, compared to the subgame perfect equilibria obtained when the community initiates integration, these heuristic metrics can effectively predict the selected profile provided that the REC adopts a financial objective, such as the net present value maximization or the total cost minimization. However, their reliability decreases if the REC follows criteria such as the minimization of carbon emissions or the price per kWh.

   b) We conduct an extensive parametric study to demonstrate the flexibility of our modeling framework. We show via simulation that the outcomes at SPEs and the behavior of stakeholders are influenced by various aspects of the problem: the order of decisions of actors, the preference criteria (or nature of the objectives) of the candidate and the REC, as well as the prospect theory parameters. More precisely, the order of decisions and stakeholders' preference criteria modify the strategies adopted, which may lead to solutions that are more focused on community or individual objectives, sometimes to the detriment of the other participant. Furthermore, the integration of prospect theory shows that stakeholder bounded rational choices introduce deviations from the behavior predicted by perfect rationality, thus impacting the final results. These deviations are mainly due to the parameters of the PT functions and, in particular, to the reference point selection method.

The remainder of the chapter is organized as follows. Section 6.2 defines the NMIPs scope and hypotheses for the REC developed in Chapter 4, as well as the decomposition of the time horizon. The theoretical presentation of extensive-form game formulations, the different preference criteria and the incorporation of prospect theory are detailed in Section 6.3. The heuristic metrics, i.e., the matching score and collective self-consumption developed by Mustika et al. [20] are described in Section 6.4. Section 6.5 presents the case-study data. Simulation results for rational stakeholders are discussed in Section 6.6, whereas Section 6 presents scenarios and outcomes of the simulations for bounded rational stakeholders. The conclusions are reported in Section 6.8.

6.2. New member integration problem (NMIP)

6.2.1. Problem scope and hypotheses

The New Member Integration Problem (NMIP) is defined for a collaborative community built on a demand-side management (DSM) scheme and composed of consumers and prosumers connected to the same LV distribution feeder (Fig. 6.1). The members can purchase their green electricity locally in the REC pool where the excess of local PV productions are mutualized, and to retail markets for gray electricity non-produced locally. This corresponds to a renewable energy community (REC) with design D2 proposed in Chapter 4. We assume that when the community was created, each member contributed with an individual investment in renewable generation as solar panels or energy storage assets. These devices are fully owned by their users, meaning that the ownership and management of each installation remains at the individual level, and without infrastructure pooling. Only surplus local renewable energy can be made available to other members.

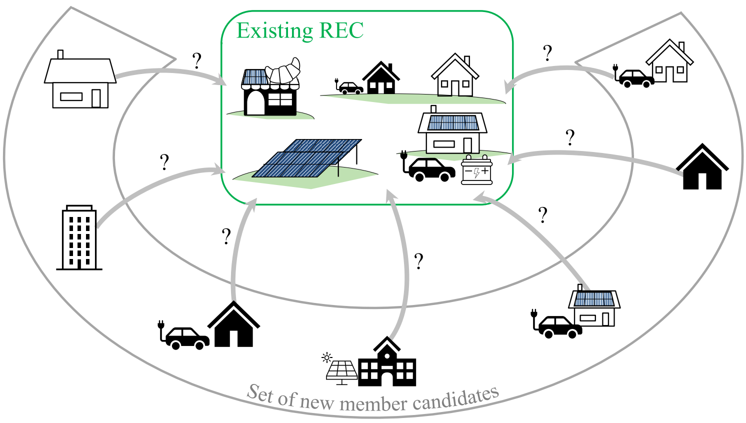

Figure 6.1.: Integration of a new member in an existing REC.

We suppose an existing renewable energy community (REC), with the set of initial members noted $\mathcal{N} = \{1, \ldots, N\}$ and the set of new member candidates as $\mathcal{M} = \{1, \ldots, M\}$. We propose two distinct approaches. In the first structure, we model the case of an external user interested in joining the community, with or without investment contribution. The second approach examines the situation where the community is the instigator of its own expansion. This analysis thus compares the dynamics of voluntary spontaneous adhesion with those of integration driven by established community members or the community manager.

In both approaches, we assume that agents choose actions which will have a significant impact on their daily operational management over the next Y years. The new member integration problem is divided into two levels to account for the different time horizons, up to Y years. The Long-Term (LT) level is associated with planning decisions with LT consequences, i.e., decisions taken at time 0 and which have a significant impact on the long run, notably on the daily operational management of energy resources. (Section 6.2.3). The Short-Term (ST) level is dedicated to the day-ahead energy resources scheduling decisions as described in Chapter4. The purpose of the NMIP is, therefore, to study the profitability and viability of long-term decisions on individual preferences, within a community framework. We initially assume that agents are interested in minimizing their total costs over the Y-year period.

6.2.2. Short-term decisions

The DSM model is formulated as a day-ahead energy resources scheduling problem reducing REC energy costs by providing an optimal allocation of resources among members. It is implemented as a convex optimization problem:

$$\min_{\Theta \in \Omega_{\mathrm{ST}}} f(\Theta), \tag{6.1}$$

where $\Omega_{\mathrm{ST}} \subseteq \mathbb{R}^n$ is the convex feasible set. The user i's decision variables set is defined as $\Theta_i = \{x_{i,a}, s_i, l_i, i_i^{com}, e_i^{com}, i_i^{ret}, e_i^{ret}, \overline{p}_i\}$ and $\Theta := (\Theta_1, \ldots, \Theta_N)$. We note $\mathcal{T} = \{1, \ldots, T\}$ the set of time steps of duration Δt for a given day. Each stakeholder $i \in \mathcal{N}$ may own shiftable appliances ($x_{i,a}$), PV panels (g_i) and battery storage system (s_i). We assume a perfect forecast of the non-flexible load (d_i) and local generation. The physical net load of member i at the time t is noted l_i^t. The user either imports, $l_i^t \geqslant 0$, or exports, $l_i^t < 0$, energy from/to the grid. We define the positive and negative net load $l_i^{t+} = \max(0, l_i^t)$ and $l_i^{t-} = \max(0, -l_i^t)$. We note $\overline{p}_i$ the peak power consumption over the day for the user $i \in \mathcal{N}$.

For billing purposes, we define virtual flow[1] variables. If the net load is positive, the energy is imported from the REC pool $i_i^{com,t}$ and/or from the supplier $i_i^{ret,t}$. Similarly, if the net load is negative, the energy surplus can be sold to other members $e_i^{com,t}$ and/or to the supplier $e_i^{ret,t}$.

The total cost minimization is subject to various constraints given the context. Every member must satisfy technical, capacity and budget requirements (we refer to Section 4.2 on page 84 for the complete development). Note the presence

[1] representing commercial, monetary-based flows.

of global constraints, which indicate that the total excess production allocated to the community must equal the total quantity imported by members at each time step.

The objective of the REC is to reduce energy costs. The community electricity bill includes commodity and grid costs. The commodity cost consists of:

- Supplier costs: these costs relate to the part of consumption not covered by local energy. For each user $i \in \mathcal{N}$, we have $C^t_{ret,i} = \lambda^t_{imp} i^{ret,t}_i$.
- Local electricity costs: these costs are associated with the energy purchased from the REC's local pool at price λ^t_{iloc}. For each user $i \in \mathcal{N}$, we have $C^t_{loc,i} = \lambda^t_{iloc} i^{com,t}_i$.
- Revenues from exported electricity: these revenues come from the sale of surplus local production. A prosumer $i \in \mathcal{N}$ earns $R^t_{ret,i} = \lambda^t_{exp} e^{ret,t}_i$, for the energy sold to the supplier, and/or $R^t_{loc,i} = \lambda^t_{eloc} e^{com,t}_i$ for the energy exported on the local REC pool.

The network costs are based on the grid (upstream transmission and distribution grids, and local distribution grid) usage. We assume that these costs correspond to the T2 pricing presented in Chapter 4:

- Volumetric-based costs: these costs are in line with the real tariffs applied in Flanders (Belgium [128]), on which the energy consumed locally can benefit from a possible discount $\gamma \in [0, 1]$. For each member $i \in \mathcal{N}$, we have $C^t_{gr,i} = \alpha(i^{ret,t}_i + \gamma i^{com,t}_i)$, with α in [€/kWh].
- Capacity-based costs: these costs are based on the peak power consumption over the day. For each user $i \in \mathcal{N}$, we have $C_{p,i} = \beta \overline{p}_i$, with β in [€/kW].

Hence, the total costs of the REC can be expressed for all profiles $\Theta \in \Omega_{\mathrm{ST}}$ as:

$$f(\Theta) = \sum_{t \in \mathcal{T}} \Big[\sum_{i \in \mathcal{N}} (C^t_{ret,i} + C^t_{loc,i} - R^t_{ret,i} - R^t_{loc,i} + C^t_{gr,i}) \Big] + \sum_{i \in \mathcal{N}} C_{p,i}. \tag{6.2}$$

In Section 4.3.2, we formulated the day-ahead resources scheduling problem as a generalized Nash equilibrium problem (GNEP). In this way, strategic interactions between members sharing common resources (energy pool and network) can be captured and the privacy-preserving properties of the associated distributed resolution algorithms can be exploited. As a reminder, in this framework each member is a selfish player who aims at minimizing his own daily cost function b_i defined by (6.4), subject to individual and global constraints. Therefore, the user's strategies set depends on the strategies of the other members: $\Omega_{\mathrm{ST},i}(\Theta_{-i})$. A member $i \in \mathcal{N}$ solves the following optimization problem, given Θ_{-i} the rivals' strategies

$$\mathcal{G} := \begin{cases} \min\limits_{\Theta_i} & b_i(\Theta_i, \Theta_{-i}) \qquad \forall i \in \mathcal{N} \\ \text{s.t.} & \Theta_i \in \Omega_{\text{ST},i}(\Theta_{-i}). \end{cases} \tag{6.3}$$

A strategy profile Θ^* is a generalized Nash equilibrium (GNE) of the game $\mathcal{G}$, if for all $i \in \mathcal{N}$ and $\Theta_i \in \Omega_{\text{ST},i}(\Theta_{-i})$, we have $b_i(\Theta^*) \leqslant b_i(\Theta_i, \Theta^*_{-i})$. See Section 3.2.4 for further details.

We assume that the total cost is distributed among community members continuously at each time step $t \in \mathcal{T}$. The billing of a member $i \in \mathcal{N}$ is defined by

$$b_i(\Theta) = \sum_{t \in \mathcal{T}} (C^t_{i,ret} + C^t_{i,loc} - R^t_{i,ret} - R^t_{i,loc} + C^t_{i,gr}) + C^t_{i,p}. \tag{6.4}$$

In the centralized optimization (6.1), the allocation is computed ex-post, whereas the cost distribution is endogenized in the members' objective functions for the GNEP (6.3).

6.2.3. Long-term decisions

The two approaches proposed for integrating a new member into a REC, present significant differences in terms of decision processes and integration perspectives. Then, the set of decisions and the order of decision-making vary according to the approach considered. These differences allow us to analyze how the flexibility or thoroughness of integration processes can influence the REC's dynamics, cost and energy stability, maintaining consistency with its own objectives and end-users' satisfaction. In the first instance, we suppose that candidate users and the community want to minimize their total costs over the next Y years. To do this, they can resort to various actions. Some of these actions require decisions to be taken at time 0, with consequences extending over the Y years, such as investment, while others are daily, as DSM. Figure 6.2 shows the complete NMIPs timeline and the different decision time horizons. We detail the NMIP in the first case considered. Note that the problem considers the short-term decisions of all REC members, whereas at the long-term level, it considers the REC as a single entity making LT integration decisions.

Case 1: new user's point of view

Let $j \in \mathcal{M}$ be a user, potentially owning PV installations of capacity $Q_{pv,j}$ kWp, who is considering the interest of being part of this REC (Year 0 on the

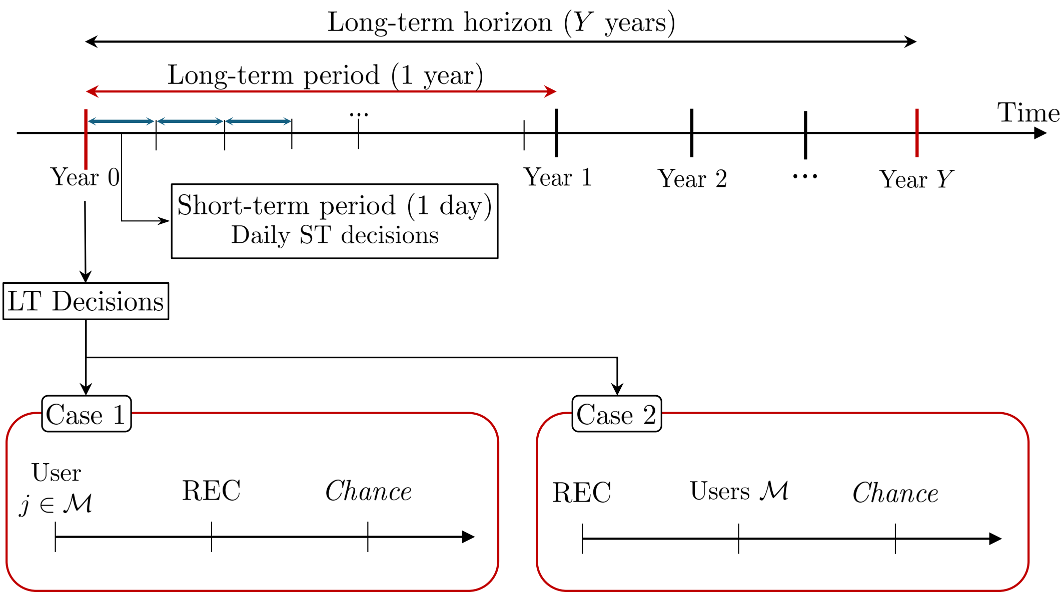

Figure 6.2.: Complete timeline of NMIPs with short-term and long-term periods defined for the various decisions.

timeline in Fig.6.2). The LT decision-making process of this stakeholder is split into two steps. In the first step, the user chooses between remaining **alone** or to **enter** the community. Next, she takes LT investment decisions among several investment profiles. For the sake of simplicity, and since the model does not attempt to determine the user's optimal investment, we assume a finite set of investment options, numbered from 1 to K, is available and noted $\mathcal{Q}$. For all $k \in \{\mathbf{1}, \ldots, \mathbf{K}\}$, we have $q_k = (q_{pv,k}, q_{st,k}) \in \mathcal{Q}$ where

- $q_{pv,k}$ is a PV capacity in [kWp], such as $q_{pv,k} \in \{0, 1, \ldots, q_{pv}^{max}\}$ with $q_{pv}^{max} \in \mathbb{Z}$.
- $q_{st,k}$ is equal to 1 if the profile includes a battery with a capacity of E^{st} in [kWh] and a maximum charging power of M^{ch} in [kW], otherwise it is 0.

We consider the first profile $k = 1$ to represent the case where the user does not make any new investments, thus $q_1 = (0, 0)$. The investment costs (CAPEX) of user $j \in \mathcal{M}$ for an investment profile $q_k \in \mathcal{Q}$ is defined as:

$$C_{inv,j} = \lambda_{pv} q_{pv,k} + \lambda_{cap} E^{st} + \lambda_{pow} M^{\text{ch}}, \tag{6.5}$$

where λ_{pv} is the photovoltaic price expressed in [€/kWp] and $\lambda_{cap}, \lambda_{pow}$ the price of the storage system linked to the energy [€/kWh] and power [€/kW] components respectively.

Once the user $j \in \mathcal{M}$ has decided to join the community with an investment option, the REC can adjust its local fees λ_{iloc}^t and λ_{eloc}^t to compensate for the impact of expanding membership. Therefore, the community can **I**ncrease, **D**ecrease, or leave **C**onstant its local prices.

In other words, it is an external user $j \in \mathcal{M}$ who triggers the integration process, investigating whether joining the REC will benefit her own objectives. The community then has the possibility of modifying its tariffs for the import and export of energy from the local pool, in order to compensate for this new user integration and her energy profile. The user j's LT decision variables set is defined as Ξ_j^1, and the one of the REC is denoted by Ξ_{rec}^1, such as $\Xi^1 = (\Xi_j^1, \Xi_{\text{rec}}^1)$. We define the feasible set of LT decisions for this approach by Ω_{LT}^1.

We integrate the uncertainty linked to long-term retail import prices λ_{imp}^t evolutions over the time horizon, through distinct scenarios with associated probabilities. For the sake of representation, this work assumes an example of a set of three scenarios $\Omega_c = \{\Psi_1, \Psi_2, \Psi_3\}$ defined as follow:

- $\boldsymbol{\Psi_1}$: The price increases slightly each year,
- $\boldsymbol{\Psi_2}$: The price increases moderately each year,
- $\boldsymbol{\Psi_3}$: A price crisis scenario.

Each scenario represents a plausible trajectory for the import price, with an associated probability (p_1, p_2, p_3). Note that the method has no restrictions on the number of possible scenarios, as long as it remains finite. It is possible to extend the problem to infinite cases, but this is not discussed here. For each scenario Ψ, we associated a function ψ that models the price λ_{imp} over the long-term horizon $\mathcal{Y} = \{1, \ldots, Y\}$.

The short-term time horizon is considered as one day. Each ST decision is an action in a ST problem, dependent on production and consumption profiles. This ST problem is solved daily over the entire horizon (see Fig.6.2). According to the actions Ξ_j^1 chosen by user j, the ST problem gathers one or two day-ahead resources scheduling models. These models are used to provide an estimate of operational costs (OPEX). Given $\Xi^1 \in \Omega_{\text{LT}}^1$, the ST problem is defined as follows:

- If Alone $\in \Xi_j^1$, then daily operations for the user j are modeled by a linear optimization problem, minimizing her own energy bill f_j without being in the community

$$f_j(\Theta_j) = \sum_{t \in \mathcal{T}} \left(C_{ret,j}^t - R_{ret,j}^t + C_{gr,j}^t \right) + C_{p,j}. \tag{6.6}$$

The REC remains with its initial composition and solves the day-ahead energy resources scheduling model in (6.3) at subsection 6.2.2.

- Otherwise, user j joins the community and can also purchase or sell surplus production from the REC pool. The REC must therefore adjust ST recommendations to reflect its new dynamism. Then, the REC solves the day-ahead energy resources scheduling problem in (6.3) for all the $N+1$ members of the set $\mathcal{N} \cup \{\text{user } j\}$, considering the new energy profile of the community.

The investment profile chosen by user j also has an impact on the short-term problem. The user has a basic energy profile, possibly with an initial PV capacity of $Q_{pv,j}$ kWp. For an investment $q_k \in \mathcal{Q}$, she will possess a total panel capacity of $Q_{pv,j} + q_{pv,k}$ kWp and/or a storage battery. This modifies her energy profile and impacts the daily consumption and production behavior of the user j. If this user joins the community, the REC's energy profile must now take into account the new member's profile and its impact on that of the original members $\mathcal{N}$. In addition, the REC can adjust local tariffs λ_{iloc} and λ_{eloc}, which also modifies the basic setting of the ST problem. Finally, the market purchase prices λ_{imp} for the day are set by the function ψ for the scenario and year in effect.

In conclusion, the short-term problem is parametrized by agents' LT decisions and the retail price evolution scenario. Given $\Xi^1 \in \Omega^1_{\text{LT}}$, $\Psi \in \Omega_c$ and a year $y \in \mathcal{Y}$, the results of the short-term problem are $\zeta_j(\Xi^1, \psi(y))$ and $\zeta_{\text{rec}}(\Xi^1, \psi(y))$, representing respectively the individual daily bill of user j and the one of the initial members of the community defined in 6.2 (even if user j has entered). The total operational cost for the Y years is simply the sum of the daily bills over the entire LT horizon $\mathcal{Y}$:

$$\zeta_j^{\text{ST}}(\Xi^1, \Psi) := \sum_{y=1}^{Y} \sum_{h=1}^{365} \zeta_j(\Xi^1, \psi(y)), \tag{6.7}$$

$$\zeta_{\text{rec}}^{\text{ST}}(\Xi^1, \Psi) := \sum_{y=1}^{Y} \sum_{h=1}^{365} \zeta_{\text{rec}}(\Xi^1, \psi(y)). \tag{6.8}$$

Then, the agents' total costs in the NMIP are defined:

$$C_{tot,j} = C_{inv,j} + \zeta_j^{\text{ST}} \tag{6.9}$$

$$C_{tot,\text{rec}} = \zeta_{\text{rec}}^{\text{ST}}. \tag{6.10}$$

Case 2: REC selects a new user

In this approach, the community is the initiator of its own expansion. The REC has a list of candidates fitting the admission criteria $\mathcal{M}$, and evaluates their profiles to identify the one that would best contribute to the stability, energy efficiency and sustainability of the community in alignment with its objectives[2]. We also consider the case where no candidate on the list is chosen. Hence, the community selects at most one user among the M-candidates. For each candidate $j \in \mathcal{M}$, the REC decides whether it **admits** user j directly, or would agree to accept user j through an **investment** in new assets.

As in the previous case, each candidate can choose from a finite set of investment options $\mathcal{Q}$ and is charged associated costs (6.5). Further, except when no one is chosen, the REC can adjust increase, decrease or leave constant local import and export fees; λ^t_{iloc} and λ^t_{eloc}. We define the LT decision variables set of a user $j \in \mathcal{M}$ and the REC as Ξ^2_j and Ξ^2_{rec} respectively, such as $\Xi^2 = (\Xi^2_1, \dots, \Xi^2_M, \Xi^2_{\text{rec}})$. We denote Ω^2_{LT} the feasible set for this approach of the problem. We also take into account the uncertainty of retail import price λ_{imp} as in the first case. As the rest of the problem description is similar to the first case, we do not discuss it further.

The NMIPs decisions for the two levels considered are shown in the Table 6.1.

Agent	Long-term decisions	Short-term decisions
User $j \in \mathcal{M}$	• **(Case 1 only)** Alone or enter • Investment profile $q_k \in \mathcal{Q}$	$\forall$ day $\in [1, 365 \times Y]$, • Flexible load ($x_{j,a}$) • Net load (l^+_j, l^-_j) • Storage (s_j) • Peak load ($\overline{p}_j$) • External exchanges (i^{ret}_j, e^{ret}_j) • Internal exchanges (i^{com}_j, e^{com}_j)
REC	• **(Case 2 only)** 0 or one user $j \in \mathcal{M}$ • **(Case 2 only)** Admitted or invest • Increase, decrease or constant $\lambda_{iloc}, \lambda_{eloc}$	$\forall i \in \mathcal{N}$, $\forall$day $\in [1, 365 \times Y]$, • Flexible load ($x_{i,a}$) • Net load (l^+_i, l^-_i) • Storage (s_i) • Peak load ($\overline{p}_i$) • External exchanges (i^{ret}_i, e^{ret}_i) • Internal exchanges (i^{com}_i, e^{com}_i)

Table 6.1.: Dispatch of NMIP long-term and short-term decisions.

[2] This paradigm may raise concerns regarding its social implications, as it could limit access for households experiencing energy poverty, potentially reducing inclusiveness within the REC. These aspects are not further discussed here.

6.3. Theoretical modeling of the NMIP

6.3.1. Extensive game formulation, analysis and resolution

Section 6.2 identifies several actions available to each candidate user $j \in \mathcal{M}$ and the REC for achieving their objectives based on their respective preferences. In addition, NMIP decisions have to be taken on different time horizons (see Table 6.1 and Fig. 6.2). The integration, investment and price adjustment decisions are made at time 0 with long-term impact, while operational decisions are made on a daily basis. The NMIP is therefore a multi-agent and multi-time horizon problem. There are many methodologies for modeling and solving such problems. Game theory describes and analyzes strategic interactions between different rational agents, who make decisions to optimize their own individual objectives. Then, we use the concept of noncooperative games to model and solve new member integration problems.

As mentioned, the timing and order of decisions are essential elements of the problem framework. However, the strategic (or normal-form) games presented in Chapters 3 and 4 are mainly static, with players choosing their strategies simultaneously. They do not allow us to represent this notion of succession in the decision-making process. In order to capture the sequential structure of the decision-making process, we represent the NMIP as an extensive-form game, as detailed in Section 5.2 of Chapter 5. We observed in subsection 6.2.3 that, depending on the approach used to handle the problem, the order and LT decisions are not the same. We therefore formulate two games, one for each approach.

Extensive games

Each game is an extensive-form game with exogenous uncertainty (Section 5.2.4) and simultaneous moves (Section 5.2.3) in the lower level, structured as a sequential decision tree. We define the game related to the NMIP with the first approach, i.e., when an external user $j \in \mathcal{M}$ initiates the integration process.

For the case 1 and a user $j \in \mathcal{M}$, the extensive game Γ_j^1 associated to the NMIP, is represented in Figure 6.3 with:

- The set of players is $\mathcal{N}_j^1 = \{\text{user } j, \text{REC}\}$, but we note that the gray diamond in Fig.6.3 corresponds to nodes where *chance* determines the action taken,
- The action set is $\Omega^1_{\text{LT},c} := \Omega^1_{\text{LT},j} \times \Omega^1_{\text{LT,rec}} \times \Omega_c$ such as
  - The action set of user j is $\Omega^1_{\text{LT},j} := \{\text{Alone, Enter}\} \cup \mathcal{Q}$,
  - The REC's actions set is $\Omega^1_{\text{LT,rec}} := \{\text{Increase, Decrease, Constant}\}$,

  - The set of actions available to chance is $\Omega_c := \{\Psi_1, \Psi_2, \Psi_3\}$.
- The set of terminal nodes of the game is noted $\mathcal{Z}^1$
- A set of functions $\varphi^1 : \mathcal{Z}^1 \to \mathbb{R}$ that associates the total costs for players at each corresponding terminal node $z \in \mathcal{Z}^1$.

Exogenous uncertainty introduces probabilities associated with the evolution of the market's electricity import price over the Y years. Each branch representing this uncertainty is marked by a probability.

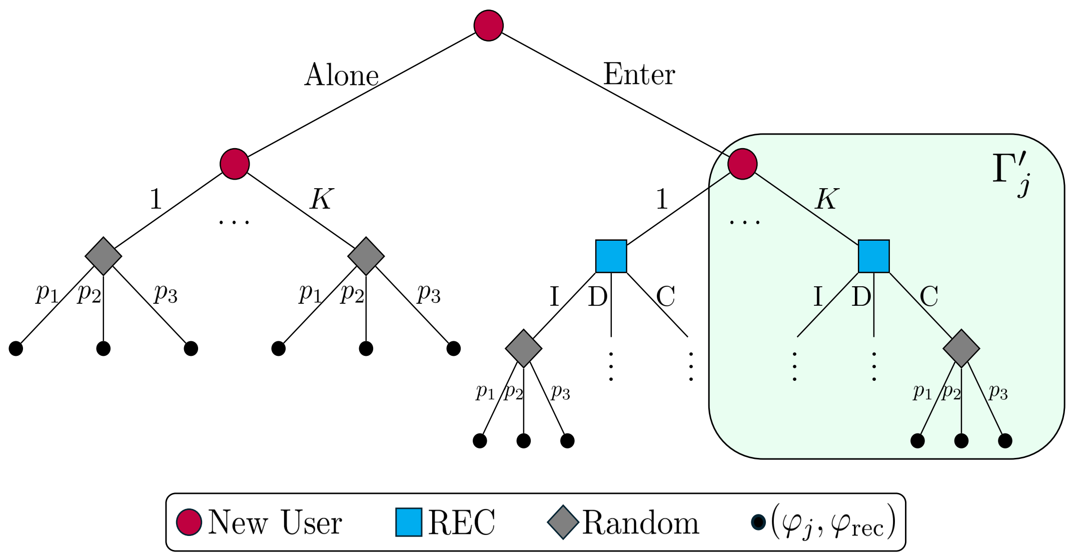

Figure 6.3.: The new member integration game with a new user's point of view.

For the remainder of this work, we do not explicitly indicate the exponent in the notations for the sake of clarity.

The terminal nodes of this tree are associated with a function φ that determines the players' total costs ((6.9)-(6.10)) over the Y years. We noted in the Section 6.2.3 that the total costs include notably operational costs for the Y-year period. They are provided by ζ^{ST}, which is actually the sum of daily invoices over the entire horizon $\mathcal{Y}$. As a reminder, the energy management decisions (third column of Table 6.1) must be taken on a ST time horizon of one day. For each day of the years studied, the daily bills are the results of the ST problem defined according to the terminal node $z \in \mathcal{Z}$ and the ongoing year $y \in \mathcal{Y}$. Thus, the user j's individual bill ζ_j is either the optimal result of an optimization problem (if Alone $\in z$), or the outcome valued at an equilibrium of a GNEP (6.3). Whereas, the total bill of the initial members of the REC ζ_{rec} is the result obtained at an equilibrium from a GNEP (6.3).

In summary, for each terminal node $z \in \mathcal{Z}$, the ST problem (GNEP and possibly optimization problem) is solved daily for each day of the time horizon Y, and the results are accumulated as each run is completed. In addition to LT decisions, ζ_j^{ST} and $\zeta_{\mathrm{rec}}^{\mathrm{ST}}$ directly influence the utilities attached to each terminal node $z \in \mathcal{Z}$:

$$\varphi_j(z) = C_{inv,j} + \zeta_j^{\mathrm{ST}}(z), \tag{6.11}$$

$$\varphi_{\mathrm{rec}}(z) = \zeta_{\mathrm{rec}}^{\mathrm{ST}}(z), \tag{6.12}$$

which represents total costs over the entire period.

We are only describing the specific aspects of the second game to avoid repetition. In the case 2, the REC evaluates a list of candidates for its expansion. The extensive game Γ^2 associated to the NMIP is displayed in Figure 6.4 with

- The set of players is $\mathcal{N}^2 = \mathcal{M} \cup \{\mathrm{REC}\}$, again the gray diamonds in Fig. 6.4 are the nodes where *chance* determines the action taken,
- The action set is $\Omega^2_{\mathrm{LT},c} := (\prod_{j\in\mathcal{M}} \Omega^2_{\mathrm{LT},j}) \times \Omega^2_{\mathrm{LT,rec}} \times \Omega_c$ such as
  - For all $j \in \mathcal{M}$, the actions set of user j is $\Omega^2_{\mathrm{LT},j} := \mathcal{Q}$ $(:= (q_k)_{k=1}^K)$,
  - The REC's set of actions is $\Omega^2_{\mathrm{LT,rec}} := \mathcal{M} \cup \{\text{No-one, Admitted, Invest, I, D, C}\}$
  - The set of actions available to chance is $\Omega_c := \{\Psi_1, \Psi_2, \Psi_3\}$.
- The set of terminal nodes of the game is noted $\mathcal{Z}^2$
- A set of functions $\varphi^2 : \mathcal{Z}^2 \to \mathbb{R}$ that associates the payoff for players at each corresponding terminal node $z \in \mathcal{Z}^2$.

The rest is similar to the case 1.

The subgame perfect equilibrium (SPE) is an appropriate solution concept for this type of game. The formal definition is set out in Definition 5.4, it is based on the idea that each player adopts an optimal strategy not only for the whole game, but also for each sub-part of it (called subgame). In other words, a strategy profile is a SPE if it induced a Nash equilibrium at every decision node of the game, regardless of the previous choices that led to that stage. In our cases, a strategy profile is a SPE if every LT decision is optimal conditionally on future decisions, while respecting the local equilibria of the GNEPs and the global minima of linear optimization problems. Remember that the result of a strategy profile is a probability distribution on the terminal nodes because of the exogenous uncertainty. Hence, players minimize their expected total costs rather than their certain invoices (see Section 5.2.4). We note SPE(Γ^1) and SPE(Γ^2) the SPEs set of the game Γ^1 and Γ^2 respectively.

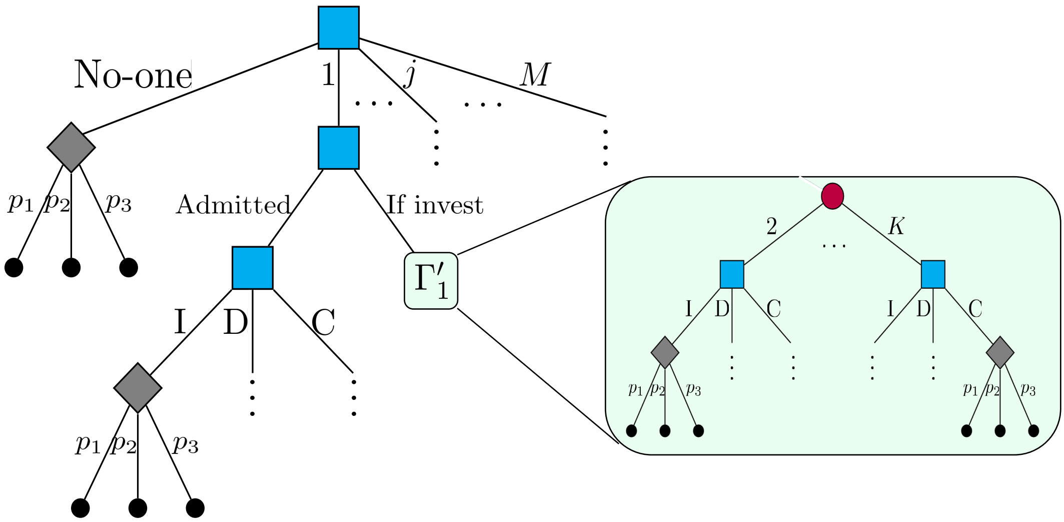

Figure 6.4.: The new member integration game when the REC selects a user.

NMIPs analysis and resolution

We study the SPE existence and resolution of the games Γ^1 and Γ^2. We begin by examining the equilibria of the short-term problem for one day.

Lemma 6.1. *Let Γ^1 and Γ^2 be extensive-form games. For each terminal node and every day for years of the total horizon, the short-term problem has at least one solution.*

Proof. Let a terminal node $z \in \mathcal{Z}$. According to $z \in \mathcal{Z}$, the daily ST problem is as follows:

- If a user $j \in \mathcal{M}$ does not joint the REC, he solves a linear optimization problem independent of the REC's actions. We are therefore guaranteed to obtain a global minimum.
- Otherwise, user j joins the community. Further, the REC solves the day-ahead energy resources scheduling problem for all the $N + 1$ members of the set $\mathcal{N} \cup \{\text{user } j\}$, considering the new energy profile of the community.

In the case of the REC with or without a new user, a GNEP must be solved. In fact, the game $\mathcal{G}$ in (6.3) corresponds to a GNEP under T2 pricing with the continuous allocation method [CB] discussed in Chapter 4 and in the Appendix B. Then, we know that our GNEP $\mathcal{G}$ has at least one generalized Nash equilibrium (GNE) by Theorem 4.4. In conclusion, we can guarantee the existence of ST solution for each representation of the ST problem. ■

The Lemma 6.1 states that for each day of the years considered and for each previous action, and scenario, there always exists a solution to the ST problem. This means that intermediate decisions and the price evolution scenario only influence the initial conditions of the ST models. They have no impact on the solutions analysis of the ST problems, which can then be solved independently. This greatly simplifies the analysis of games SPEs.

The optimal solutions from successive ST problem solving are then included in the cost functions φ via ζ^{ST} in (6.11) and (6.12). For each user $j \in \mathcal{M}$, the function ζ_j^{ST} in (6.7), corresponds to the sum of operational costs minimized (if the user is alone) or evaluated at GNE (if the user joins the REC), over the complete horizon. For the community, $\zeta_{\mathrm{rec}}^{\mathrm{ST}}$ in (6.8) is the accumulated operational costs of the REC's initial members $\mathcal{N}$ valued at a GNE, over the period. Thereby, the games Γ^1 and Γ^2 turn out to be finite extensive games with perfect information conditional on the realization of uncertainty. Consequently, we can establish the following result through Kuhn's Theorem 5.1.

Theorem 6.1. *Given extensive-form games with chance and simultaneous moves Γ^1 in Fig. 6.3 and Γ^2 in Fig.6.4. There always exists a Subgame Perfect Equilibrium.*

Actually, we can provide more details about the GNEs of GNEPs. In fact, we have a jointly convex GNEP (see Definition 3.20), for which we can characterize the set of variational equilibria VE($\mathcal{G}$) (see Definition 3.25), a subset of GNEs. Thanks to Theorem 4.5.4 and Corollary 4.2.2, we can establish that the set of variational equilibria coincides with the social optimum set of the centralized optimization problem (6.1). Thus, each VE is a social optimum, and finding a VE is equivalent to calculating a global minimum of (6.1). So for each day of the year under consideration, we can be sure that an efficient GNE can be reached, i.e., a GNE that corresponds to a social optimum. So, if the REC has no new members, it is possible to find a strategy profile inducing a VE for each day. This means that the payoff function of the REC for the process $\zeta_{\mathrm{rec}}^{\mathrm{ST}}$ corresponds to the sum of the community's optimal daily costs.

Short-term problems can be solved independently for each terminal node $z \in \mathcal{Z}$ and each price scenario $\psi \in \Psi$. We can use standard algorithms to perform linear optimization problems [94, 129]. Given the configuration of a GNEP $\mathcal{G}$, we have from the Theorem B.2 on page 225, the sequence generated by the Proximal Decomposition Algorithm (PDA) with shared constraints, converges to a VE of the game $\mathcal{G}$. Assuming that duplicates are not counted, a number of models can be solved depending on the game studied:

1. For the extensive game Γ_j^1, given a user $j \in \mathcal{M}$,

- The number of linear optimization problems is $365 \times Y \times 3 \times K$,
- The number of GNEPs is $365 \times Y \times 3 \times (1 + (K \times 3))$.

2. For the extensive game Γ^2,
   - The number of linear optimization problems is $365 \times Y \times 3 \times M$,
   - The number of GNEPs is $365 \times Y \times 3 \times (1 + (M \times K \times 3))$.

After this step, SPEs can be obtained through the backward induction procedure for both games (see Sections 5.2.2 and 5.2.4). It is important to point out that with this methodology, backward induction does not obtain all the SPEs of the games, but rather the SPEs set conditioned by the VEs and social optima obtained in the simultaneous nodes!

6.3.2. Modeling the heterogeneous preference criteria of actors

A large body of literature dedicated to local markets focuses on minimizing participants' costs. However, focusing solely on this aspect does not capture the full range of motivations that drive end-users to participate in a REC. Indeed, the growing end-user awareness of environmental challenges tends to suggest that end-users may adopt decisions that do not fully optimize their total costs, but which are more in line with their ecological principles, including other concerns such as social, reliability, etc. It would also overlook other benefits of RECs, which can provide environmental and social benefits as well as economic ones to the REC's members. Furthermore, end-users may exhibit differences in terms of their energy preferences within a single REC. To better represent this diversity and heterogeneity of motivations, we propose that external users and community preferences can adopt different criteria or objectives, driving their long-term decisions.

Remark 6.1. We consider the community as a single entity when making long-term decisions; thus we only regard the preferences of the REC as a whole. Future research could consider the study of all the individual preferences of each member. In addition, the REC evaluates each criterion in respect of its original members, even if a new user has joined the community.

Four criteria based on financial indicators as well as one environmental criterion are identified. As an illustration of the impact of the LT decision criterion on games, we use the extensive game Γ^1_j in Fig.6.3 for a user $j \in \mathcal{M}$ (bearing in mind that it's the same for the other game).

1. **Total costs.** The benchmark case involves minimizing the total costs over the time horizon. The aggregate operating costs for the considered

period are noted as C_{op}. The total costs are

$$C_{tot} := C_{inv} + C_{op}. \tag{6.13}$$

Note that for the REC, we look at the total operational costs of the REC without the new member to make the LT decisions, then there are no investment costs and $C_{op,\text{rec}} = \sum_{i\in\mathcal{N}} C_{op,i}$. This is our basic assumption, so in our game Γ^1_j, the functions φ_j and φ_{rec} are defined by (6.11) and (6.12) respectively.

2. **Net Present Value (NPV)**. This criterion maximizes the LT profitability of energy investments by integrating the time values of money. The NPV is the discounted sum of all cash-flows associated with an investment project over a period Y. For each user $j \in \mathcal{M}$

$$\begin{aligned} \text{NPV}_j &:= -C_{inv,j} + \sum_{y=1}^{Y} \frac{C_j^{y+} - C_j^{y-}}{(1+\kappa)^y} \\ &= -C_{inv,j} - \sum_{y=1}^{Y} \frac{C_{op,j}^{y}}{(1+\kappa)^y} \end{aligned} \tag{6.14}$$

where $C_{inv,j}$ is the investment costs, C_j^{y+} and C_j^{y-} are the positive and negative financial flows respectively; and κ is the discount rate. In our game Γ^1_j, we can consider that each division in the sum of the equation (6.14) is the result ζ_j of the ST problem. Thus, ζ_j^{ST} is the sum over the entire LT period $\mathcal{Y}$. Let $z \in \mathcal{Z}$, the payoff function of user j is defined:

$$\varphi_j(z) = -C_{inv,j} - \underbrace{\sum_{y=1}^{Y}\sum_{h=1}^{365} \underbrace{\frac{C_{op,j}^{y,h}}{(1+\kappa)^y}}_{\zeta_j(z,y)}}_{\zeta_j^{\text{ST}}(z)}, \tag{6.15}$$

with the day $h \in \{1,\dots,365\}$. Note that the equation is similar for the REC considering only its original members, but with the investment costs equal to 0.

3. **Return on Investment (ROI)**. This criterion is applied specifically to new users $\mathcal{M}$, aiming to maximize the ROI on investments made in the integration process. It is the ratio between net income (over a given period) and the investment costs. A high ROI means that investment gains compare favorably with investment costs. It is a way of relating

profits to capital invested. For each user $j \in \mathcal{M}$

$$\mathrm{ROI}_j = \frac{\text{Net profit of user } j}{C_{inv,j}}. \tag{6.16}$$

The net profit generated by the investment corresponds to the difference between user j's operational costs in the initial case (i.e., Alone and without investment q_1) and those obtained in the case studied $z \in \mathcal{Z}$. Note that we are comparing the elements for the same price scenario realization. Hence, let $\Psi \in \Omega_c$, $z_0 =$(Alone, q_1, Ψ), and $z \in \mathcal{Z}$ such as $\Psi \in z$, the payoff function of user j is:

$$\varphi_j(z) = \frac{\zeta_j^{\mathrm{ST}}}{C_{inv,j}} = \frac{1}{C_{inv,j}} \sum_{y=1}^{Y} \sum_{h=1}^{365} (\zeta_j(z_0, y) - \zeta_j(z, y)). \tag{6.17}$$

4. **Carbon Dioxide Emissions (CDE)**. This criterion aims to minimize carbon dioxide emissions associated with electrical unit consumption. The daily CDE of the initial members of the REC (even if user j decides to join) is defined as follows:

$$\mathrm{CDE} := \gamma_{pv}^{CO2} \Big(\sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{N}} i_i^{com,t} \Big) + \gamma_{mix}^{CO2} \Big(\sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{N}} i_i^{ret,t} \Big), \tag{6.18}$$

where γ_{pv}^{CO2} and γ_{mix}^{CO2} measuring greenhouse gas emissions from photovoltaic electricity generation and the national energy mix respectively in [gCO2eq/kWh]. We then consider that the result ζ_{rec} of a daily ST problem corresponds to (6.18) evaluated at equilibrium. So, for all $z \in \mathcal{Z}$,

$$\varphi_{\mathrm{rec}}(z) = \zeta_{\mathrm{rec}}^{\mathrm{ST}}(z) = \sum_{y=1}^{Y} \sum_{h=1}^{365} \zeta_{\mathrm{rec}}(z, y). \tag{6.19}$$

The reasoning is similar for user j. However, if Alone$\in z$, then all the i_j^{com} are equal to zero and i_j^{ret} actually correspond to l^+ in (6.18).

5. **Price per kWh [PkWh]**. This criterion aims to reduce the cost of energy per kilowatt-hour to keep energy affordable and accessible to all members

$$\mathrm{PkWh} := \frac{\text{Total operational costs over the entire horizon}}{\text{Positive net charge over the entire horizon}}. \tag{6.20}$$

Given the REC, we need to retrieve two daily results in the ST problem. We have $\zeta_{\text{rec}} = (\zeta_{\text{rec},1}, \zeta_{\text{rec},2})$ with $\zeta_{\text{rec},1}$ the total cost and $\zeta_{\text{rec},2}$ the aggregated net positive load of the REC members. Then, for $z \in \mathcal{Z}$, the REC's payoff is defined by

$$\varphi_{\text{rec}}(z) = \frac{\sum_{y=1}^{Y} \sum_{h=1}^{365} \zeta_{\text{rec},1}(z, y)}{\sum_{y=1}^{Y} \sum_{h=1}^{365} \zeta_{\text{rec},2}(z, y)}. \tag{6.21}$$

The case is similar for the user j.

Through the use of these criteria, our method aims to express the economic and environmental motivations of the existing REC, while taking into account the specific preferences of potential newcomers.

Remark 6.2. Heterogeneity in player preferences only occurs at the global level of NMIPs. For the ST problem, the objective functions used in linear optimization problems and GNEPs remain the same for all players. They consist of minimizing daily total bills. Thus, individual preferences do not modify the dynamics of daily decisions, but they do influence the cumulative results associated with the terminal nodes of the tree, integrating long-term results.

6.3.3. Modeling the bounded rational behaviour of actors using Prospect Theory

Currently, most of the literature adopts the hypothesis of the rationality of economic agents, and relies more generally on the expected utility theory (EUT) [8]. However, this theory fails to predict an individual's real decision-making when facing uncertainty. Kahneman and Tversky have proposed the Prospect Theory (PT) as an alternative way of modeling a bounded rational individual's decision-making in the presence of uncertainty. We refer to the Section 5.3 in Chapter 5 for a fuller introduction.

In this work, we apply PT only for the long-term decision-making under retail import price uncertainty. As a reminder, LT decisions involve an analysis of cumulative outcomes associated with the terminal nodes of the tree. In this framework, end-users and RECs are faced with uncertainty over several years, and must consider different alternatives according to their preferences and their perceptions of risks and probabilities. We assume that short-term decisions are taken rationally, and that LT decisions solely may exhibit bounded rational behaviors. This is first justified by the fact that most of ST decisions may be computed and implemented by a controller without human intervention.

Furthermore, the ST problem is mainly used to model the operational costs over the whole horizon, so that the influence on LT outcomes of possible bounded rational behaviors in ST decisions is a priori limited. This justifies the methodological separation from LT studies.

Although we represent the REC as a single entity to simplify long-term modeling, its decisions remain the product of interaction between the members and possibly a community manager, each with their own preferences, objectives and biases. These interactions, combined with often complex collective decision-making procedures (e.g., voting, consensus, delegation) could induce features of bounded rationality on a collective scale. Thus, it would not be astonishing if a REC could adopt behaviors analogous to those observed in individuals. As a result, we consider that the REC can be modeled as a boundedly rational entity and justify the PT framework.

To illustrate this approach, we consider the first case, where a user $j \in \mathcal{M}$ is planning to join an energy community, represented by the extensive-form game Γ^1 in Fig.6.3. We also assume that both the user j and the community, want to minimize their total costs (6.13). Once again, for the sake of clarity, we no longer mention the exponent. Let $\Xi \in \Omega_{\mathrm{LT}}$, we define the terminal nodes set in the subgame $\Gamma_{|\Xi}$ as

$$\mathcal{Z}_{|\Xi} := \{z \in \mathcal{Z} \mid \Xi \sqsubseteq z\} . \tag{6.22}$$

The basic formulation of PT for assessing the global value of an alternative $z \in \mathcal{Z}_{|\Xi}$, given $\Xi \in \Omega_{\mathrm{LT}}$, is a combination of two elements: a subjective value function (Section 5.3.3) and a probability weighting function (Section 5.3.4), which we summarize in the following.

According to PT, each player perceives an outcome as a gain or loss relative with respect to an individual reference point. This is called the framing effect. An outcome is regarded as a gain if it is greater than the reference point, while it is perceived as a loss if it is smaller than the reference point. Hence, each player has a subjective value function, capturing the subjective perception of outcomes given the reference point. The subjective value function $v : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ of a player, established in [81] is given by

$$v(-\varphi(z), r) := \begin{cases} ((-\varphi(z)) - r)^{\eta_a} & \text{if } (-\varphi(z)) \geqslant r \\ -\eta_c(r - (-\varphi(z)))^{\eta_b} & \text{if } (-\varphi(z)) < r, \end{cases} \tag{6.23}$$

with $\varphi(z)$ the total cost at the terminal node $z \in \mathcal{Z}$, r is the reference point, the coefficients η_a and η_b captured the diminishing sensitivity effect, and the coefficient η_c reflects the player's loss aversion.

Remark 6.3. For preference criteria to be maximized, such as (6.14), (6.16) and (6.20), the subjective value function directly evaluates the outcome $\varphi(z)$ without the minus.

In this study, the individual reference point corresponds to the initial situation before any decision is taken. It reflects the current state in terms of energy asset ownership, the cost associated with their operations, and any other relevant variables. We investigate the impact of the reference point choice on players' strategic behavior in two cases:

1. **Fixed reference point**. The reference point is determined as the worst possible outcome among the three import retail prices scenarios Ω_c for the initial situation, and denoted r^{max}.
2. **Stochastic reference point**. The reference point varies according to the scenario under consideration, reflecting fluctuations in the costs of the initial situation linked to the uncertainty of the import price. This approach, noted r^{stoc}, was introduced in [167] and has already been applied to energy investment problems [150].

The second element of PT is the probability weighting effect. In contrast to EUT, in which probabilities are evaluated objectively, experiments reveal that individuals tend to overestimate low probability and underestimate high probabilities. The weighting function reflects these distorted perceptions by assigning a subjective decision weight to each objective probability. Based on experimental results, the probability weighting function $w : [0,1] \to [0,1]$ given in [81] is mathematically formulated by

$$w(p) := \frac{p^{\eta_d}}{(p^{\eta_d} + (1-p)^{\eta_d})^{\frac{1}{\eta_d}}}, \tag{6.24}$$

where p is the objective probability of the scenario $\Psi \in \Omega_c$, and η_d is the probability weighting parameter.

Therefore, the prospect theory equation to calculate the global value of an alternative $\mathcal{Z}_{|\Xi}$, given $\Xi \in \Omega_{\mathrm{LT}}$, is defined as

$$V(\mathcal{Z}_{|\Xi}) := \sum_{\epsilon=1}^{3} w(p_\epsilon) v(\varphi(z_\varepsilon), r). \tag{6.25}$$

In Section 6.7, we compare results from the application of PT with the base case where players are assumed to be rational in the sense of EUT. The EUT postulates that rational players make decisions which maximize their expected utility. Then, the expected utility of an alternative $\mathcal{Z}_{|\Xi}$, given $\Xi \in \Omega_{\mathrm{LT}}$, is defined as

$$U(\mathcal{Z}_{|\Xi}) := \sum_{\epsilon=1}^{3} p_\epsilon u(\varphi(z_\varepsilon)), \tag{6.26}$$

with $u : \mathbb{R} \to \mathbb{R}$ the player's utility function. We assume that each player is not biased by risks, such as $u(\varphi(z_\epsilon)) = \varphi(z_\epsilon)$.

6.4. Benchmark model

We present here the heuristic method based on two performance metrics, established by Mustika et al. in [20] to discriminate new member candidates to joint an existing REC. The framework studied by the authors seems at first sight to be in line with our NMIP (see case 2 in Section 6.2.3). The authors distinguish two different status for the existing REC: the need for more production or consumption and the need for more storage assets.

The two proposed metrics are the matching score energy (MS) and the collective self-consumption energy (CSC), which are used to investigate the need for production and consumption. They are calculated at each time step $t \in \mathcal{T}$ on a day $h \in \{1, \dots, 365\}$, and the sum over the Y years corresponds to the total contribution.

6.4.1. Matching score

This metric relies on the so-called "community mismatch profile" (CMP). For a day h, on each time step $t \in \mathcal{T}$, the CMP is calculated from the net energy balance of the REC, either a surplus of a deficit.

$$\mathrm{CMP}^t_{\mathrm{rec}} = \sum_{i \in \mathcal{N}} g^t_i - \sum_{i \in \mathcal{N}} \Big(d^t_i + \sum_{a \in \mathcal{A}_i} x^t_{i,a} \Big). \tag{6.27}$$

The CMP is developed to facilitate the comparison of the net load curve of new member candidates in $\mathcal{M}$. For each candidate $j \in \mathcal{M}$, the net load curve of the user j is defined as the candidate's consumption minus production at each time step. This is given by:

$$\mathrm{P}^{net-load,t}_j = d^t_j + \sum_{a \in \mathcal{A}_j} x^t_{j,a} - g^t_j. \tag{6.28}$$

A scoring system is introduced to evaluate the matching degree between the REC's needs and the users' profile (via Algorithm 1 in [20]). For a user $j \in \mathcal{M}$

and every $t \in \mathcal{T}$ of a day h, we note this score as MS_j^t. Given $\mathrm{CMP}_{\mathrm{rec}}^t$ and $\mathrm{P}_j^{net-load,t}$, we have

- If the REC has a production deficit ($\mathrm{CMP}_{\mathrm{rec}}^t < 0$) and the user j has a surplus ($\mathrm{P}_j^{net-load,t} < 0$), then the score corresponds to the production excess ($\mathrm{MS}_j^t = \mathrm{P}_j^{net-load,t}$);
- If the REC has an excess of production ($\mathrm{CMP}_{\mathrm{rec}}^t > 0$) and the user j is lacking energy ($\mathrm{P}_j^{net-load,t} > 0$), then the score is the value of the net load ($\mathrm{MS}_j^t = -\mathrm{P}_j^{net-load,t}$);
- Otherwise, $\mathrm{MS}_j^t = 0$.

The contribution of the candidate $j \in \mathcal{M}$ is founded on the total score at each time step $t \in \mathcal{T}$, accumulated for each day of the Y years considered:

$$\mathrm{MS}_j = \sum_{y=1}^{Y} \sum_{h=1}^{365} \sum_{t=1}^{T} \mathrm{MS}_j^t. \tag{6.29}$$

6.4.2. Collective self-consumption

The second metric relies on the collective self-consumption energy. It is defined as the part of the REC's consumption covered by local generation, as defined by

$$\mathrm{CSC} = \sum_{y=1}^{Y} \sum_{h=1}^{365} \sum_{t \in \mathcal{T}} \left(\min \left(\sum_{i \in \mathcal{N}} g_i^t, \sum_{i \in \mathcal{N}} \left(d_i^t + \sum_{a \in \mathcal{A}_i} x_{i,a}^t \right) \right) \right). \tag{6.30}$$

The CSC energy is affected by the addition of a new member j, depending on this user's energy profiles. Consequently, the update CSC is calculated as

$$\mathrm{CSC}_j' = \sum_{y=1}^{Y} \sum_{h=1}^{365} \sum_{t \in \mathcal{T}} \left(\min \left(\sum_{i \in \mathcal{N}} g_i^t + g_j^t, \sum_{i \in \mathcal{N}} (d_i^t + \sum_{a \in \mathcal{A}_i} x_{i,a}^t) + d_j^t + \sum_{a \in \mathcal{A}_j} x_{j,a}^t \right) \right). \tag{6.31}$$

Hence, the user j's contribution $\Delta\mathrm{CSC}_j$ is measured as the difference between the existing REC's CSC energy before and after the integration of this new member.

$$\Delta\mathrm{CSC}_j = \mathrm{CSC}_j' - \mathrm{CSC}. \tag{6.32}$$

6.4.3. Battery energy storage

Beyond the demand for increased energy production or consumption in an established energy community, the integration of storage systems can also prove interesting for managing the balance between local energy production and use. We describe the approach used in [20] to assess battery requirements.

The need for batteries in the existing REC depends on the amount of energy that can be stored to enhance independence from the main grid (i.e., to increase the self-sufficiency ratio). The additional battery requirement, noted by $\Delta E_{\text{rec}}^{bat}$ in [kWh], is defined as the minimum daily average energy between the REC's surplus and deficit, considering the usable storage capacity of the existing battery systems. We have

$$\Delta E_{\text{rec}}^{bat} = \max\left(0, \Big(\frac{\min(\overline{E_{\text{rec}}^{sur}}, \overline{E_{\text{rec}}^{def}})}{\Delta\text{SOC}}, -E_{\text{rec}}^{bat,init}\Big)\right), \tag{6.33}$$

with ΔSOC the battery's real usability factor (generally 0.9-0.2=0.7). The daily average energy surplus in the REC $\overline{E_{\text{rec}}^{sur}}$ is calculated as the total surplus from each member:

$$\overline{E_{\text{rec}}^{sur}} = \frac{\sum_{y=1}^{Y}\sum_{h=1}^{365}\sum_{t\in\mathcal{T}} \max\left(0, \sum_{i\in\mathcal{N}}(g_i - d_i^t - \sum_{a\in\mathcal{A}_i} x_{i,a}^t)\right)}{365 \times Y}. \tag{6.34}$$

In addition, the daily average energy deficit in the REC $\overline{E_{\text{rec}}^{def}}$ is computed as the total deficit from each member:

$$\overline{E_{\text{rec}}^{def}} = \frac{\sum_{y=1}^{Y}\sum_{h=1}^{365}\sum_{t\in\mathcal{T}} \max\left(0, \sum_{i\in\mathcal{N}}(d_i^t + \sum_{a\in\mathcal{A}_i} x_{i,a}^t - g_i)\right)}{365 \times Y}. \tag{6.35}$$

Finally, the contribution made by a user $j \in \mathcal{M}$ is based on her battery capacity and the needs of the REC:

$$S_j^{bat} = \min(\Delta E_{\text{rec}}^{bat}, E_j^{bat}). \tag{6.36}$$

Lastly, we combine the value contributions from both production/consumption and the battery system:

$$S_j^{\text{MS}} = \text{MS}_j + 365 \times Y \times S_j^{bat}, \tag{6.37}$$
$$S_j^{\text{CSC}} = \Delta\text{CSC}_j + 365 \times Y \times S_j^{bat}. \tag{6.38}$$

In this work, we assume that the candidates in $\mathcal{M}$ do not initially own a storage battery. So, for each user $j \in \mathcal{M}$, we have $E_j^{bat} = 0$. As $\Delta E_{\text{rec}}^{bat}$ is always greater than or equal to 0 by definition (6.33), we thus have that $S_j^{bat} = 0$ in (6.36).

6.5. Case-study

We study various instances on the NMIP which are simulated on a representative year replicated through a 20-year horizon ($Y = 20$). The basic year is built on 8 representative days, i.e., 1 week day and 1 weekend day for each season. We study the NMIPs for three renewable energy communities, each composed of five members connected behind the same MV-LV feeder. The purpose is to represent communities with various electrical profiles in order to simulate the NMIP in different situations. These profiles include:

- A REC with an annual energy deficit (see Table 6.2), where consumption significantly exceeds local production,
- A REC with an annual energy surplus (see Table 6.3), where production exceeds consumption,
- An energy-balanced REC (see Table 6.4), where production and consumption are approximately equal on an annual basis. For the sake of concision, we do not present the results of this community in Sections 6.6 and 6.7.

Remark 6.4. Note that we have also simulated communities of 55 members, similarly as in Section 4.5. However, for facilitating the analysis, we present here communities with 5 members.

Further, a set of 11 candidates is also considered with varying profiles in terms of production capacity ($Q_{pv,j}$ kWp), energy consumption and flexibility. They are displayed in Table 6.5. Two profiles stand out among them. Candidate profile 7 represents a pure supplier profile, without any consumption. On the other hand, candidate profile 11 was specifically designed for the community in surplus (Table 6.3). This profile only includes non-flexible consumption that aligns perfectly with the time steps during which the community experiences surplus production. Hence, this candidate should theoretically be able to her entire energy needs using the REC's surplus alone. While this makes profile 11 an ideal candidate, it is not a realistic one.

	PV (kWp)	ESS	Prod. (MWh)	Non-flex. cons. (MWh)	Flex. cons. (MWh)	Total cons. (MWh)	Flex. level (%)	Cons.-prod. (MWh)
1	0	1	0	3.131	11.88	15.012	112.7	15.012
2	3	1	3.511	3.131	0.720	3.852	149.5	0.341
3	2	0	2.340	7.532	9	16.532	54.44	14.191
4	0	0	0	1.084	2.232	3.316	67.31	3.316
5	0	0	0	5.473	6.552	12.025	54.5	12.025
REC	5	2	5.851	20.351	30.384	50.735	79.75	44.884

Table 6.2.: Composition and annual energy profile of a 5-member REC in deficit.

	PV (kWp)	ESS	Prod. (MWh)	Non-flex. cons. (MWh)	Flex. cons. (MWh)	Total cons. (MWh)	Flex. level (%)	Cons.-prod. (MWh)
1	20	1	23.406	3.131	11.88	15.012	112.7	-8.393
2	4	1	4.681	3.131	0.720	3.852	149.5	-0.829
3	9	1	10.532	4.992	1.44	6.432	100.7	-4.101
4	15	0	17.553	1.084	2.232	3.316	67.31	-14.237
5	10	0	11.702	5.473	6.552	12.025	54.5	0.322
REC	58	3	67.873	17.811	22.824	40.635	93.38	-27.238

Table 6.3.: Composition and annual energy profile of a 5-member REC in surplus.

	PV (kWp)	ESS	Prod. (MWh)	Non-flex. cons. (MWh)	Flex. cons. (MWh)	Total cons. (MWh)	Flex. level (%)	Cons.-prod. (MWh)
1	14	1	16.383	3.131	11.88	15.012	112.7	-1.372
2	4	1	4.681	3.131	0.720	3.852	149.5	-0.829
3	9	1	10.532	4.992	1.44	6.432	100.7	-4.101
4	0	0	0	1.084	2.232	3.316	67.31	3.316
5	10	0	11.702	5.473	6.552	12.025	54.5	0.322
REC	37	3	43.298	17.811	22.824	40.635	93.38	-2.663

Table 6.4.: Composition and annual energy profile of a 5-member balanced REC.

User	PV (kWp)	ESS	Prod. (MWh)	Non-flex. cons. (MWh)	Flex. cons. (MWh)	Total cons. (MWh)	Flex. level (%)	Cons.-prod. (MWh)
1	0	0	0	25.98	0	25.98	0	25.98
2	10	0	11.702	1.084	11.88	12.964	91.64	1.262
3	0	0	0	1.084	0	1.084	0	1.084
4	10	0	11.702	25.98	9	34.98	25.73	23.278
5	0	0	0	5.473	6.552	12.025	54.5	12.025
6	1	0	1.17	9.933	8.352	18.285	45.68	17.115
7	20	0	23.405	0	0	0	0	-23.405
8	7	0	8.192	2.89	5.112	8.002	63.88	-0.19
9	4	0	4.681	2.907	0.720	3.627	19.85	-1.054
10	5	0	5.851	2.383	10.44	12.823	81.41	6.972
11	0	0	0	46.928	0	46.928	0	46.928

Table 6.5.: Annual profiles of new user candidates.

For each user candidate $j \in \mathcal{M}$, there are $K = 22$ LT investment options available. They can choose to invest in solar panels with a capacity ranging from 0 kWp to 10 kWp, in 1 kWp increments and/or also install a domestic battery with a capacity of 14 kWh and a power setting of 5 kW. As a reminder, we also consider that a user may not invest (q_1). The photovoltaic price is fixed at $\lambda_{pv} = 1500$€/kWp, while the tariffs for the ESS are $\lambda_{cap} = 300$€/kWh and $\lambda_{pow} = 300$€/kW. The community can adjust its local tariffs λ_{iloc} and λ_{eloc}, as

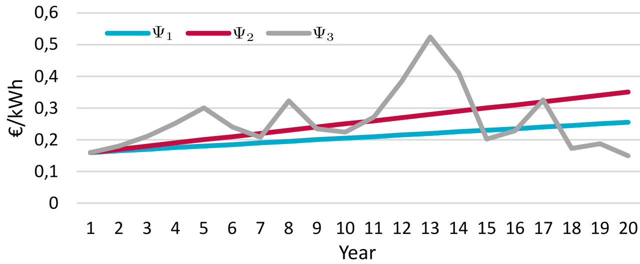

Figure 6.5.: Import retail price evolution.

part of its LT decision-making process. Three options are available: increase or decrease the local tariffs by 0.01€/kWh or keeping them constant.

We assume three scenarios for the evolution of retail import prices over the next 20 years (see Fig. 6.5):

- $\mathbf{\Psi_1}$: The price increases by 0.005 €/kWh each year,
- $\mathbf{\Psi_2}$: The price increases by 0.01 €/kWh each year,
- $\mathbf{\Psi_3}$: The price follows the electricity price trends on the Belgian market during the energy crisis[3]

with their associated probability $p_1 = 1/6$, $p_2 = 1/3$ and $p_3 = 1/2$.
Thus, the tree of the extensive game Γ^1_j in Fig. 6.3 is made up of 264 terminal nodes ($|\mathcal{Z}^1| = 264$) for each user $j \in \mathcal{M}$. For the extensive game Γ^2 in Fig.6.4, we have 2181 terminal nodes ($|\mathcal{Z}^2| = 2181$).

For the non-flexible loads , hourly electricity consumption profiles are extracted from the Pecan Street Project dataset [131] and generated for whole days, with $T = 24$. We assume these profiles remain unchanged over the simulation. Community members and candidate users may own different flexible devices: dishwashers, washing machines, clothes dryers, electrical vehicles and heat pumps. For the heat pumps, different energy needs are applied according to the season. The initial and final battery state-of-charge are fixed at 50%. We consider bi-hourly commodity tariffs: $\lambda^t_{imp} = 0.08$ €/kWh (at year 0), $\lambda^t_{exp} = 0.02$ €/kWh, $\lambda^t_{iloc} = 0.065$ €/kWh and $\lambda^t_{eloc} = 0.032$ €/kWh between 21pm and 4am, and $\lambda^t_{imp} = 0.16$ €/kWh (at year 0), $\lambda^t_{exp} = 0.04$ €/kWh, $\lambda^t_{iloc} = 0.13$ €/kWh and $\lambda^t_{eloc} = 0.05$ €/kWh elsewhere. We assume constant network tariffs with $\alpha = 0.027$ €/kWh and a discount of $\gamma = 50\%$, and $\beta = 0.11$ €/kW.

In Section 6.3.1, we restated that any variational equilibrium (VE) of the

[3]based on formulas used to calculate Engie's import tariffs [168], which vary month by month with the EPEX DAM index [169].

GNEP, calculated for the ST problem, corresponds to a social optimum. So, every VE of the game is efficient. Furthermore, for this specific billing structure, we observed numerically in Chapter 4 that the individual bills obtained with the GNEP and via the ex-post distribution of a social optimum show only marginal differences (with a maximum deviation of 1.8% in Table 4.6 on page 113 (D2, tariff T2 and [CB])). As a result, performing the centralized optimization problem is equivalent to finding the VEs of the GNEP. Therefore, we solve the ST problem using a classical centralized algorithm, treating it as a convex centralized optimization problem to speed up the computational process. The distributed algorithm argument is not really necessary in this case, as we are looking at long-term scenarios to define a new user's investment (the REC members do not necessarily need to reveal their real operational data). Given a user $j \in \mathcal{M}$, to build the extensive game Γ^1_j, we need to solve $20 \times 8 \times 3 \times (22 + 1 + (22 \times 3)) = 42\ 720$ optimization problems, whereas the extensive game Γ^2 requires solving $20 \times 8 \times 3 \times (1 + 11 + (11 \times 22 \times 3)) = 354$ 240 optimization problems.

Short-term problems are coded in the open source language Julia 1.8 [22] using the JuMP package, and solved using Gurobi [23]. Extensive games are implemented and analyzed using Python 3.11.6 [21] with the nutree library. The trees terminal nodes store the players' gains or costs, calculated from the short-term simulations, according to the specified preference criterion. In order to determine subgame perfect equilibria, we implemented the backward induction method: starting from the terminal nodes, optimal strategies are calculated back to the root. The case study is conducted on an Intel(R) Core(TM) i7-1260P 2.10 GHz with 325 Go of RAM. The whole range of short-term simulations for a REC and a candidate user from $\mathcal{M}$, were solved with a maximum time of 147.20s.

We have simulated various combinations of preference criteria, which are described in Section 6.3.2: total costs (6.13), NPV (6.14), CO2 emissions (6.18), ROI (6.16) and the price per kWh (6.20). For the NPV, we consider two cases, one with an discount rate of κ_1=4.14% (Statbel July 2023 [170]), denoted NPV1, and another with a rate of κ_2=0.36% (Statbel October 2023 [170]), noted NPV2. We assume that candidate users adhere to the same preference. Thus, the heterogeneity arises between the candidates set $\mathcal{M}$ and the community. As a summary, we tested 25 combinations of extensive games and users $j \in \mathcal{M}$. For clarity, we do not present the results of every combination. Instead, we focus on describing the most relevant cases.

	Deficit	Surplus
NPV1	-144 253.34€	-43 524.35€
NPV2	-212 943.88€	-65 448.51€
C_{tot}	221 640.703€	68 231.05€
CDE	137.564 tCO2eq	69.016 tCO2eq
PkWh	0.239€/kWh	0.147€/kWh

Table 6.6.: Expected utilities U_{rec} of existing communities without new members for each criterion.

6.6. Results for perfectly rational agents

In this section, we analyze the results obtained from the simulations under the assumption that all agents are perfectly rational in the sense of the expected utility theory (Section 5.3.1). This establishes a baseline for evaluating the decisions made in our models. Subsection 6.6.1 observes whether the result obtained by the heuristic methods advanced by Mustika et al. [20], can help to predict the new member selected by the REC in the NMIP formulated by the extensive game Γ^2 (Fig. 6.4). The impact of the order of decision-making in the NMIP on SPEs and stakeholders' behavior, is studied in subsection 6.6.2. A comparison is presented between extensive games Γ^2 (Fig. 6.4) and Γ^1 (Fig. 6.3), from the point of view of the candidates.

6.6.1. Heuristic vs equilibrium

The results of the heuristic methods proposed by Mustika et al. in [20] are shown and compared with the SPEs' outcomes of the extensive game Γ^2 in Fig. 6.4, when it comes to selecting a potential candidate for the RECs studied. As a reminder, this game models the NMIP where the community aims to expand by integrating a new member from a list of potential candidates $\mathcal{M}$. We start by establishing homogeneous criteria before simulating heterogeneous criteria. For each metric (MS and CSC) described in Section 6.4, the contribution values are normalized by dividing each user's contribution by the maximum valued observed, so as to clearly distinguish the performance of each method [20].

REC in deficit

We study the case of the REC in an annual energy deficit situation as represented by Table 6.2. The CSC energy of the existing REC with five members is 5.677 MWh, which we take as a reference to calculate the improvement in CSC energy with an additional candidate $j \in \mathcal{M}$ in the community. The Table 6.7 shows the candidate results for both metrics, together with their normalized values. The shaded boxes indicate the maximum value for each metric. The normalized value for each candidate user of $\mathcal{M}$ with MS and CSC methods, is displayed in Figure 6.6.

User	MS	ΔCSC (kWh)	Normal MS	Normal CSC
1	1 142.54	173.77	0.062	0.016
2	5 298.12	6 568.99	0.288	0.595
3	34.46	34.37	0.002	0.003
4	887.61	11 044.25	0.048	1
5	205.54	124.72	0.011	0.011
6	309.79	1 315.37	0.017	0.119
7	18 423	5 085.56	1	0.46
8	2 439.304	6 161.7	0.132	0.558
9	2 549.72	2 705.02	0.138	0.245
10	1 701.17	4 735.54	0.092	0.429
11	10 753.93	173.77	0.584	0.016

Table 6.7.: Net and normalized MS and CSC values of potential candidates in $\mathcal{M}$, for the energy-deficit REC.

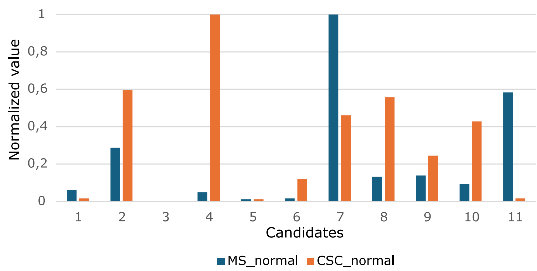

Figure 6.6.: Normalized values for both metrics for each user in $\mathcal{M}$ and the energy-deficit REC.

Results differ depending on the metric. The rankings in descending order of MS and CSC, are given by

$$\text{MS}_{\text{def}} : \{7, 11, 2, 9, 8, 10, 1, 4, 6, 5, 3\}, \tag{6.39}$$

$$\text{CSC}_{\text{def}} : \{4, 2, 8, 7, 10, 9, 6, 1, 11, 5, 3\}. \tag{6.40}$$

Profiles 1, 3, 5 and 11 are consumers only, and we can see that the value provided by these candidates when using MS is higher than with CSC in this case. Profiles 2, 4, 8 and 10 had a large amount of individual self-consumption, which is considered in the CSC metric, but not in the MS. User 4 is still a heavy consumer (see Table 6.5). In terms of both metrics, users 5 and 3 are suitable in their current state for this REC. Since profile 7 is a large producer, it is pretty obvious that it is the one that is going to get the biggest MS in the situation where the community is short of energy. He can make her production available to other members of the community, but it may be advantageous to export some of it to retail markets.

We investigate the results of the extensive game Γ^2 in Fig. 6.4. In fact, across all tested combinations of preference criteria, including heterogeneous ones, the REC always selects candidate 7, while other actions vary depending on the preferences. This result is not surprising, given that user 7 represents a pure producer profile. This observation is in line with expectations, suggesting that the model does indeed seem to identify the most logical and optimal choice under these conditions, validating its behavior to some extent. Given the particular nature of the profile 7, we have decided to remove it from the list of candidates for the remainder of the analysis: $\mathcal{M}' = \mathcal{M} \backslash \{7\}$. Readers interested in the full set of results, including candidate 7, can refer to the Appendix C.1.

We first analyze the results of the extensive game Γ^2 in Fig. 6.4 with the candidate list $\mathcal{M}'$ by considering scenarios in which all agents adopt the same preference criteria. In particular, the maximization of NPV (6.14), the minimization of total costs (6.13) or the reduction of CDE (6.18). These homogeneous scenarios enable direct comparison with previously calculated heuristic methods, while highlighting specific characteristics of subgame perfect equilibria in this context. Given the uncertainty associated with retail import prices evolution, the results presented in Table 6.8 correspond to the expected utilities U in (6.26) of rational agents.

	Criteria		Num. SPE	User	PV (kWp)	Stor.	Local prices	Exp. utility user	Exp. utility REC
	Users $\mathcal{M}$	REC							
1	NPV1	NPV1	1	2	+0	0	D	-21 071.6€	-134 319€
2	NPV2	NPV2	1	2	+0	0	D	-31 328.9€	-197 895.7€
3	C_{tot}	C_{tot}	1	2	+0	0	D	32 628.88€	205 942.1€
4	CDE	CDE	1	2	+10	+1	D	18.556 tCO2eq	109.65 tCO2eq

Table 6.8.: Expected utilities of SPEs obtained for the NMIP Γ^2 of the REC in deficit with the candidates set $\mathcal{M}'$ and for homogeneous preference criteria. The rationality of agents is assumed.

In Table 6.8, we observe that for each combination, there is only one SPE, and the "User" column shows that the REC selects user 2. The community directly accepts user 2 without additional investment and reduces its local prices, in the case of financial combinations 1-3. In case 4, if user 2 has more production and a storage battery, he will be able to increase her self-consumption or inject surplus into the community pool, thus reducing the CO2 linked to retail imports. It is therefore in the community's interest to demand more investment. According to both metrics (6.39) and (6.40), user 2 emerges as the most likely choice after candidate 7. Then, the selection of user 2 is in line with the expectations given the characteristics of the profile (see Tab. 6.5) and aligns with the underlying principles of heuristic methods.

We consider scenarios where the candidates set $\mathcal{M}'$ and the REC prioritize different preferences. Table 6.9 provides the full results, capturing the diversity of selected actions across the tested combinations of preferences. Despite this variety, candidate 2 remains the preferred choice in all cases due to its profile (Table 6.5).

	Criteria		Num.	User	PV	Stor.	Local	Exp. utility	Exp. utility
	Users $\mathcal{M}$	REC	SPE		(kWp)		prices	user	REC
1	NPV1	NPV1	1	2	+0	0	D	-21 071.6€	-134 319€
2	NPV2	NPV2	1	2	+0	0	D	-31 328.9€	-197 895.7€
3	C_{tot}	C_{tot}	1	2	+0	0	D	32 628.88€	205 942.1 €
4	CDE	CDE	1	2	+10	+1	D	18.556 tCO2eq	109.65 tCO2eq
5	ROI	(financial)	1	2	+0	0	D	0%	
6	ROI	CDE	1	2	+2	0	I	0.764%	119.91 tCO2eq
7	NPV1	PkWh*	243	2	+1	0	I	-20 007.2€	0.216€/kWh
8	NPV2	PkWh*	243	2	+0	0	I	-28 982.94€	0.219€/kWh
9	C_{tot}	PkWh*	243	2	+0	0	I	30 191,1€	0.219€/kWh
10	NPV1	CDE	1	2	+2	0	I	-20 565.1€	119.91 tCO2eq
11	NPV2	CDE	1	2	+2	0	I	-29 240.79€	119.91 tCO2eq
12	C_{tot}	CDE	1	2	+0	0	C	31 410.6€	123.56 tCO2eq
13	CDE	(financial)	1	2	+10	+1	D	18.556 tCO2eq	
14	PkWh*	(financial)	3	2	+10	+1	D	0.12 €/kWh	
15	PkWh*	CDE	6	2	+9	0	I	0.115€/kWh	110.073 tCO2eq

Table 6.9.: Expected utilities of SPEs obtained for the NMIP Γ^2 of the REC in deficit with the candidates set $\mathcal{M}'$ and the rationality of agents is assumed. The (financial) notation indicates that the results are valid if one of the three criteria: NPV1, NPV2 and C_{tot} is used. We have used the lexicographical order in the case of PkWh.

In order to avoid a combinatorial explosion, we used the lexicographic order (Definition 5.5) for agents with the PkWh criterion (6.20). Despite the lexicographical order on the REC, we still obtain 243 SPEs if users have one of the financial criteria (NPV1, NPV2 and C_{tot}) and the community minimizes the price per kWh (combinations 7, 8 and 9). Nevertheless the results and actions remain the same for each SPE. We can see that the expected values of users 2 are better than in the homogeneous cases (1, 2 and 3). We also note the difference between the NPV1 and NPV2 cases, confirming that the discount rate has an impact on the decision-making process. When the community minimizes the CDE (i.e., combinations 10, 11 and 12), the user 2's expected utilities are better than in the homogeneous case (1, 2 and 3), but not as good as compared to the case where the community follows the PkWh criterion. For scenario 12 in particular, the distinction with case 3 is explained by the fact that the REC keeps its prices constant. Given the lexicographical order for the users, we have 3 SPEs with the same result and actions, where candidate 2 invests in 10 kWp and a battery in combination 14. Similarly in case 15, we observe 6 SPEs with the same result and actions, but user 2 invests only in 9 kWp and her expected value is better than for the previous case.

At first glance, the outcomes of our models and the rankings based on the metrics seem to align, but we are faced with a case where a particular candidate is suitable for all combinations.

REC in surplus

We proceed in a similar way for the REC in an annual energy surplus situation as represented by Table 6.3 and with a CSC energy of 13.663 MWh. The Table 6.10 shows the candidate results for both metrics and their normalized values. Again, the shaded boxes indicate the maximum value for each metric. The maximum value for both metrics is associated with candidate 11, so the normalized values for each candidate user of $\mathcal{M}$, presented in Figure 6.7, are based on those of profile 11 for each method.

User	MS	ΔCSC (kWh)	Normal MS	Normal CSC
1	7 499.2	7 308.96	0.16	0.156
2	1.496	4 778.64	$(3.2).10^{-5}$	0.102
3	310.14	310.14	0.007	0.007
4	1 101.4	11 717.93	0.023	0.25
5	1 651.15	1 651.15	0.035	0.035
6	2 413.2	3 583.42	0.051	0.076
7	590.4	563.66	0.013	0.012
8	0.81	5 109.64	$(1.73).10^{-5}$	0.109
9	14.92	1 252.2	0.00032	0.027
10	24.89	3 856.68	0.00053	0.082
11	46 927.7	46 927.7	1	1

Table 6.10.: Net and normalized MS and CSC values of potential candidates in $\mathcal{M}$, for the energy-surplus REC.

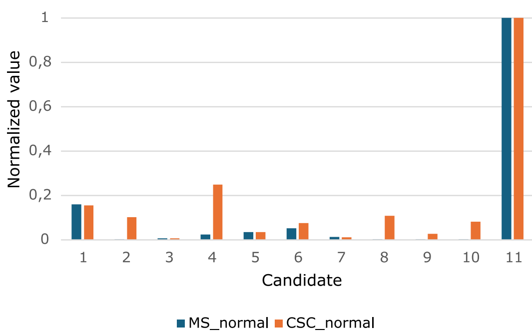

Figure 6.7.: Normalized values for both metrics for each user in $\mathcal{M}$ and the energy-surplus REC.

Then, the rankings in descending order of MS and CSC, are provided by

$$\text{MS}_{\text{sur}} : \{11, 1, 6, 5, 4, 7, 3, 10, 9, 2, 8\}, \tag{6.41}$$
$$\text{CSC}_{\text{sur}} : \{11, 4, 1, 8, 2, 10, 6, 5, 9, 7, 3\}. \tag{6.42}$$

User 11 is clearly distinguished for the energy surplus community. In fact, profile 11 was custom-built for the REC in energy surplus. Then, it only has non-flexible consumption that coincides with time steps when the community has excess production. It should therefore be able to cover all the needs with the entire REC surplus pool. Profile 1 is a large consumer without flexibility. Nevertheless, part of her needs are calibrated to moments of surplus, resulting in a high MS. If we were to exclude candidate 11, user 1 would surely be the best candidate to select.

Once again, profile 3 appears unsuitable according to the metrics. This position in the rankings could be explained by comparison with other users with significantly higher consumption value (Tab. 6.5). For this REC, the MS calculation process mechanically favors profiles with high consumption (see Section 6.4.1), giving these users a higher MS than profile 3. In addition, user 3's low demand also leads to lower self-consumption than the others. A possible solution to improve the performance could be to invest in a battery. This observation raises an important question: is it relevant to compare all users with each other? An alternative approach would be to define categories to better reflect the specific characteristics of each profile.

As for the REC in energy deficit, profiles 2, 4, 8 and 10 show a high level of individual self-consumption, taken into account in the CSC, but not in the MS. Actually, these users contribute slightly to the collective effort to consume the REC's surplus. Given the definition of CSC (see Section 6.4.2), we can conclude that these candidates provide an artificial high contribution to the REC's CSC. This somehow biases the ranking (6.42).

We examine the results for the extensive game Γ^2 in Fig. 6.4. We have found that the REC systematically selects producer 7 to minimize its CDE. Otherwise, it selects user 11, tailor-made to meet the REC's specific needs in a surplus situation. These results confirm that our model yields consistent and expected actions in terms of user selection. However, due to the particular characteristics of these two profiles, we decided to remove them from the list of candidates for the remainder of the analysis: $\overline{\mathcal{M}} = \mathcal{M} \backslash \{7, 11\}$. Note that the complete table of combinations with the initial set $\mathcal{M}$ can be found in Appendix C.2.

As in the previous case, we first consider the results of the extensive game Γ^2 in Fig. 6.4 with the candidate list $\overline{\mathcal{M}}$ by considering homogeneous combinations of preference criteria. We assume the rationality of the users and the REC, so the actions and expected utilities U (6.26), related to the SPEs, are presented in Table 6.11.

There is only one SPE for each homogeneous combination in 6.11. The REC directly includes user 1 without additional investment and increases its local prices, in the case of financial combinations 1-3. In the CDE case, we have the same set of actions as found with the energy deficit REC at Tab. 6.8. After excluding profile 11, both heuristic methods (6.41) and (6.42) identify user 1, as it stands, as the most suitable candidate to join the community. However, the metrics were unable to predict model's selection of user 2, given that neither

	Criteria Users $\mathcal{M}$	Criteria REC	Num. SPE	User	PV (kWp)	Stor.	Local prices	Exp. utility user	Exp. utility REC
1	NPV1	NPV1	1	1	+0	0	I	-77 573.5€	-42 231.2€
2	NPV2	NPV2	1	1	+0	0	I	-113 682.02€	-63 680.4€
3	C_{tot}	C_{tot}	1	1	+0	0	I	118 248.66€	66 403.65€
4	CDE	CDE	1	2	+10	+1	D	18.556 tCO2eq	68.554 tCO2eq

Table 6.11.: Expected utilities of SPEs obtained for the NMIP Γ^2 of the REC in surplus with the candidates set $\overline{\mathcal{M}}$ and for homogeneous preference criteria. The rationality of agents is assumed.

metric incorporates CO2 emissions.

This difference in users selection also encourages further study of heterogeneous preference combinations. The complete array of actions and expected utilities at SPEs for each combination, is provided in Table 6.12. Note that to avoid combinatorial explosion of backward induction, we use the lexicographical order (Definition 5.5) for the individual with the PkWh* (6.20).

	Criteria Users $\mathcal{M}$	Criteria REC	Num. SPE	User	PV (kWp)	Stor.	Local prices	Exp. utility user	Exp. utility REC
1	NPV1	NPV1	1	1	+0	0	I	-77 573.5€	-42 231.2€
2	NPV2	NPV2	1	1	+0	0	I	-113 682.02€	-63 680.4€
3	C_{tot}	C_{tot}	1	1	+0	0	I	118 248.66€	66 403.65€
4	CDE	CDE	1	2	+10	+1	D	18.556 tCO2eq	68.554 tCO2eq
5	ROI	(financial)	1	1	+0	0	I	0%	
6	ROI	CDE	1	2	+0	+1	C	0.415%	68.786 tCO2eq
7	(financial)	PkWh*	1	∅	/	/	/	/	0.146€/kWh
8	NPV1	CDE	1	2	+0	+1	C	-21 553.8€	68.786 tCO2eq
9	NPV2	CDE	1	2	+0	+1	C	-29 291.5€	68.786 tCO2eq
10	C_{tot}	CDE	1	2	+0	+1	C	30 272.4€	68.786 tCO2eq
11	CDE	(financial)	1	1	+0	0	I	60 561 tCO2eq	
12	PkWh*	(financial)	1	1	+0	+1	I	0.214€/kWh	
13	PkWh*	CDE	1	2	+10	+1	D	0.12€/kWh	68.554 tCO2eq

Table 6.12.: Expected utilities of SPEs obtained for the NMIP Γ^2 of the REC in surplus with the candidates set $\overline{\mathcal{M}}$ and the rationality of agents is assumed. The (financial) notation indicates that the results are valid if one of the three criteria: NPV1, NPV2 and C_{tot} is used. We have used the lexicographical order in the case of PkWh.

We have in Table 6.12 that each time the REC aims to minimize its CDE, it selects user 2 with investment. The candidate 2 invests in at least one battery regardless of her criteria. An intriguing outcome occurs for combination 7: the community decides not to select any candidate on the list. Note that this is an action resulting from a SPE obtained with the lexicographic order assumed for the REC. It is therefore possible that other SPEs, leading to different results, have been ignored.

Observations summary

We summarize the observations arising from the comparison between the results of heuristic methods and those of subgames perfect equilibria of the NMIP Γ^2 (Fig.6.4), where the community moves first.

1. The characteristics of preferences and their combinations directly influence the outcome of the problem, highlighting the importance of strategic choices for each stakeholder.
2. For each preference criterion of the candidates set, if the REC adopts a strictly financial preference (such as NPV1, NPV2 or total cost C_{tot}), heuristic methods can help forecast the new member selected by the community.
3. When the community pursues objectives that are not purely financial such as minimizing the CDE or the price of kWh, heuristic methods are not always able to anticipate the REC's choice. In such cases, decisions depend on the specific features of the SPEs and on the candidates' preferences.

We have also raised some points to consider in the metrics of the heuristic methods. Some candidates who consume part of their own initial production make an artificial contribution to the REC's CSC, which can distort the ranking. The case of profile 3 suggests that comparing all candidates collectively may not be the most appropriate approach.

6.6.2. Impact of the decisions order

We continue our analysis, still assuming the perfect rationality of all stakeholders. We now study the impact of the order of decision-making on equilibria and outcomes. We adopt the point of view of a user $j \in \mathcal{M}$ and compare the results obtained in the first extensive game Γ_j^1 (user j moves first) in Fig.6.3 with those obtained in the second game Γ^2 in Fig.6.4 (REC moves first). Given the size of the work, we focus on the users who were selected in a SPE of the game Γ^2 at the previous subsection 6.6.1, excluding special profiles 7 and 11.

REC in deficit

As previously observed, for each combination of preferences, the energy-deficit REC (in Tab.6.2) always selected user 2 as the new member. So to assess the impact of the order of decision-making, we compare the results obtained for this user in the two games frameworks. Table 6.13 shows the actions and expected utilities U in (6.26), associated with the SPEs from the extensive game Γ_2^1, while Table 6.9 covers the results obtained in Γ^2.

	Criteria		Num.	In	PV	Stor.	Local	Exp. utility	Exp. utility
	User 2	REC	SPE	REC	(kWp)		prices	user 2	REC
2'	NPV2	NPV2	1	1	+0	+1	D	-29 316.5€	-200 652.46€
3'	C_{tot}	C_{tot}	1	1	+0	+1	D	30 298.4€	208 822€
4'	CDE	CDE	2	0 1	+10	+1	\ D	18.556 tCO2eq	137.564 tCO2eq 109.65 tCO2eq
5'	ROI	(financial)	2	0 1	+0	+1	\ D	0.409%	
7'	NPV1	PkWh*	1	1	+0	0	I	-19 453.4€	0.219€/kWh
8'	NPV2	PkWh*	1	1	+0	+1	I	-27 456.6€	0.221€/kWh
9'	C_{tot}	PkWh*	1	1	+0	+1	I	28 366.51€	0.221€/kWh
10'	NPV1	CDE	1	1	+0	0	C	-20 262.8€	123.556 tCO2eq
12'	C_{tot}	CDE	1	1	+0	+1	D	30 298.4€	126.527 tCO2eq
13'	CDE	(financial)	2	0 1	+10	+1	\ D	18.556 tCO2eq	
14'	PkWh*	(financial)	2	0 1	+10	+1	\ D	0.12€/kWh	

Table 6.13.: Expected utilities of SPEs obtained for the NMIP Γ_2^1 of the REC in deficit where the rationality of agents is assumed. The (financial) notation indicates that the results are valid if one of the three criteria: NPV1, NPV2 and C_{tot} is used. We have used the lexicographical order in the case of PkWh.

Since the outputs are identical for preference combinations 1, 6, 11 and 15 in Tab. 6.9, for both games, we do not include them in Tab. 6.13. At first glance, the user 2 seems more inclined to invest in a battery when he makes the decision first, rather than in game Γ^2 (see combinations 2', 3', 5', 8', 9' and 12'). We observe two SPEs for combinations 4', 5', 13' and 14'. In these conditions, user 2 is indifferent between staying alone or joining the community. In addition, they present the same actions and outcomes as in the second game when the user decides to integrate the REC, with the exception of case 5'. It appears that, for all combinations of preferences, the value of the REC's expected utility is always less than or equal to that observed in Table 6.9, but greater than it would have been if the REC had chosen not to pick any users (see Table 6.6). So, although these strategies are not necessarily optimal, they still offer an advantage to the community.

REC in surplus

In the case of the energy-surplus community (in Tab. 6.3), we found earlier that the REC could choose either user 1, user 2 or to take no candidate at all. So, we focus on the profiles 1 and 2.

User 1. The Table 6.14 presented the actions and user 1's expected utilities at SPEs from the extensive game Γ^2. It is important to note that the community results, shown in the last column, represent the values achieved if the REC selected user 1! The shaded boxes indicate the SPEs where the REC actually chooses this user at SPEs of the game Γ^2 (Tab. 6.12). In comparison, Table 6.15 reports the outcomes when solving the extensive game Γ_1^1, where user 1 moves first.

	Criteria		Num.	PV	Stor.	Local	Exp. utility	Exp. utility
	Users $\mathcal{M}$	REC	SPE	(kWp)		prices	user	REC
1	NPV1	NPV1	1	+0	0	I	-77 573.5€	-42 231.2€
2	NPV2	NPV2	1	+0	0	I	-113 682.02€	-63 680.4€
3	C_{tot}	C_{tot}	1	+0	0	I	118 248.66€	66 403.65€
4	CDE	CDE	1	+10	+1	D	48 846 tCO2eq	69.183 tCO2eq
5	ROI	(financial)	1	+0	0	I	0%	
6	ROI	CDE	1	+0	0	D	0%	70.401 tCO2eq
7	(financial)	PkWh*	1	+8	+1	D		0.186€/kWh
8	NPV1	CDE	1	+7	+1	D	-71 233.513€	69.183 tCO2eq
9	NPV2	CDE	1	+8	+1	C	-96 849.48€	69.183 tCO2eq
10	C_{tot}	CDE	1	+10	+1	D	100 008.54€	69.183 tCO2eq
11	CDE	(financial)	1	+0	0	I	60 561 tCO2eq	
12	PkWh*	(financial)	1	+0	+1	I	0.214€/kWh	
13	PkWh*	CDE	1	+0	+1	D	0.207€/kWh	69.105 tCO2eq

Table 6.14.: Expected utilities of user 1 at the SPEs from NMIP Γ^2 with the REC in surplus and the candidates set $\overline{\mathcal{M}}$. The rationality of agents is assumed. The (financial) notation indicates that the results are valid if one of the three criteria: NPV1, NPV2 and C_{tot} is used. We have used the lexicographical order in the case of PkWh.

The outputs are identical for preference combinations 4, 7, 8, 9, 10, 12 and 13 (in Tab. 6.14) for both games, we do not include them in Tab. 6.15. The results can differ between the two games, regardless of whether the community selects user 1 or not. User 1 always has an interest in joining the community in both games. Furthermore, he also shows a tendency to invest more when he is the first to make a decision. The different situations observed are as follows:

	Criteria		Num.	In	PV	Stor.	Local	Exp. utility	Exp. utility
	User 1	REC	SPE	REC	(kWp)		prices	user 2	REC
1'	NPV1	NPV1	1	1	+8	+1	I	-71 533.4€	-43 263.4€
2'	NPV2	NPV2	1	1	+8	+1	I	-97 070.7€	-65 070.8€
3'	C_{tot}	C_{tot}	1	1	+9	+1	I	100 268€	67 903.6€
5'	ROI	(financial)	2	1	+1	0	I	14.505%	
6'	ROI	CDE	2	1	+1	0	D	16.288%	70.457 tCO2eq
11'	CDE	(financial)	1	1	+10	+1	I	48.846 tCO2eq	

Table 6.15.: Expected utilities of SPEs obtained for the NMIP Γ_1^1 of the REC in surplus where the rationality of agents is assumed. The (financial) notation indicates that the results are valid if one of the three criteria: NPV1, NPV2 and C_{tot} is used. We have used the lexicographical order in the case of PkWh.

- The community chooses user 1 in the game Γ^2 (Tab. 6.14), then either
  - User 1 adopts the same actions in the game Γ_1^1, as in 12,
  - Even if the actions in the game Γ_1^1 are not optimal for the REC, they are still advantageous for the community, as in 1', 2', 3', 5' and 11'.
- In both games, there are configurations where the REC has no interest in integrating user 1, as in 4, 6, 6', 7, 8, 9, 10 and 13.

User 2. The Table 6.16 shows the actions and user 2's expected utilities at SPEs from the extensive game Γ^2. Once again, the REC outcomes, shown in the last column, represent the values achieved if the REC selected user 2! The shaded boxes indicate the SPEs where the REC actually chooses this user at SPEs of the game Γ^2 (Tab. 6.12). Further, Table 6.17 indicated the outcomes for the extensive game Γ_2^1, where user 2 is the first to take an action.

For preference combinations 2, 3, 6, 9 and 10 (shown in Tab. 6.16), the results of the two games are equivalent, which is why these cases are not listed in Tab. 6.17. On the other hand, variations may appear depending on whether or not the community chooses candidate 2. The different situations identified can be summarized as follows:

- In both games, one of the stakeholders will be better off staying alone and not investing (in the case of the user 2). These are combinations 1, 1', 8 and 8'; given user 2's initial NPV1 value of -21 175.02€ and the REC's initial values in Tab. 6.6.
- User 2 chooses the same investment option in both games, further either:
  - This user is indifferent between staying alone or joining the REC, for the latter, either

	Criteria		Num.	PV	Stor.	Local	Exp. utility	Exp. utility
	Users $\mathcal{M}$	REC	SPE	[kWp]		prices	user	REC
1	NPV1	NPV1	1	+0	+1	D	-21 570.3€	-43 413.1€
2	NPV2	NPV2	1	+0	+1	D	-29 316.4€	-65 274.8€
3	C_{tot}	C_{tot}	1	+0	+1	D	30 298.4€	68 049.3€
4	CDE	CDE	1	+10	+1	D	18 556 tCO2eq	68.554 tCO2eq
5	ROI	(financial)	1	+0	+1	D	0.409%	
6	ROI	CDE	1	+0	+1	C	0.415%	68.786 tCO2eq
7	(financial)	PkWh*	1	+0	+1	I		0.156€/kWh
8	NPV1	CDE	1	+0	+1	C	-21 553.8€	68.786 tCO2eq
9	NPV2	CDE	1	+0	+1	C	-29 291.5€	68.786 tCO2eq
10	C_{tot}	CDE	1	+0	+1	C	30 272.4€	68.786 tCO2eq
11	CDE	(financial)	1	+10	+1	D	18.556 tCO2eq	
12	PkWh*	(financial)	1	+10	+1	D	0.12€/kWh	
13	PkWh*	CDE	1	+10	+1	D	0.12€/kWh	68.554 tCO2eq

Table 6.16.: Expected utilities of user 2 at the SPEs from NMIP Γ^2 with the REC in surplus and the candidates set $\overline{\mathcal{M}}$. The rationality of agents is assumed. The (financial) notation indicates that the results are valid if one of the three criteria: NPV1, NPV2 and C_{tot} is used. We have used the lexicographical order in the case of PkWh.

	Criteria		Num.	In	PV	Stor.	Local	Exp. utility	Exp. utility
	User 2	REC	SPE	REC	[kWp]		prices	user 2	REC
1’	NPV1	NPV1	1	1	+0	0	I	-20 928.7€	-43 573.3€
4’	CDE	CDE	2	0 1	+10	+1	\ D	18.556 tCO2eq	69.016 tCO2eq 68.554 tCO2eq
5’	ROI	(financial)	2	0 1	+0	+1	\ D	0.409%	
7’	NPV1	PkWh*	1	1	+0	0	C	-20 846.6€	0.16€/kWh
8’	NPV1	CDE	1	1	+0	0	I	-20 928.7€	69.214 tCO2eq
11’	CDE	(financial)	2	0 1	+10	+1	\ D	18.556 tCO2eq	
12’	PkWh*	(financial)	2	0 1	+10	+1	\ D	0.12€/kWh	
13’	PkWh*	CDE	2	0 1	+10	+1	\ D	0.12€/kWh	69.016 tCO2eq 68.554 tCO2eq

Table 6.17.: Expected utilities of SPEs obtained for the NMIP Γ_2^1 of the REC in surplus where the rationality of agents is assumed. The (financial) notation indicates that the results are valid if one of the three criteria: NPV1, NPV2 and C_{tot} is used. We have used the lexicographical order in the case of PkWh.

    * The community chooses user 2 in game Γ^2, as in 4' and 13';
    * There is a better candidate, but the user 2 strategy still benefits the community, as in 5', 11' and 12'.
  - User 2 has every interest in joining the community, for the latter, either
    * The community chooses user 2 in game Γ^2, as in 6, 9 and 10:
    * There is a better candidate, but the user 2 strategy still benefits the community, as in 2 and 3.
- In both games, the community has no interest in accepting candidate 2, who would be disadvantaged by the investment profile required by the REC in game Γ^2, as combinations 7'.

This also highlights the fact that the discount rate used to calculate the net present value (6.14), can have a significant impact on agents' behavior.

Observations summary

This subsection explored the impact of decision order in NMIP on strategies, subgame perfect equilibria and decision-maker behavior. We summarize the conclusions established by comparing the actions and outcomes obtained in game Γ^2 (Fig. 6.4) and game Γ^1_j (Fig. 6.3) equilibria, for each combination of preference criteria.

1. In the NMIP, the order of decisions influences the equilibrium and behavior of stakeholders. Depending on whether the community or a user chooses first, the agents anticipate the reactions of the other participants differently, which modifies the optimal strategies and therefore the SPEs, as well as the interests of the stakeholders.
2. Preference criteria and their parameters (e.g., the discount rate influencing the net present value (NPV)), play a decisive role in the decision-making process. These combinations of preferences can lead to differences in actions and results for the community and users.

These conclusions underline the importance of the decision-making sequential and preference criteria in the modeling of strategic interactions of energy communities and external end-users on the new member integration problem.

6.7. Results with prospect theory

In this section, we explore the long-term consequences of decisions made in the NMIP under the assumption of bounded rationality. This analysis complements the previous study (Section 6.6) by comparing the results obtained in the case of perfect rationality with those obtained from simulations using Prospect Theory [77, 81], presented in Section 6.3.3, and Section 5.3.2 in Chapter 4.

Prospect theory involves identifying two functions: a utility or subjective value function v (6.23) and a probability weighting function w (6.24), capturing the perceived probabilities. In order to carry out this evaluation, we consider different parameterizations of stakeholders' PT functions. This theory was originally developed by Kahneman and Tversky to model end-user behavior. They estimated in the experimental paper [81], a set of parameter values that are widely used: $\eta_a = \eta_b = 0.88$, $\eta_c = 2.25$ and $\eta_d = 0.65$ (PT1 in Table 6.18). However, recent empirical works have suggested that PT can also be applied to corporate levels, with alternative parameterizations, for instance $\eta_a = \eta_b = 0.45$, $\eta_c = 1.96$ and $\eta_d = 0.65$ in [143] (PT2 in Table 6.18). To our knowledge, no study has investigated so far the experimental estimation of PT parameters in the context of energy communities. This task remains, however, out of the scope of the present thesis work. We therefore test the two aforementioned parameter sets in order to provide a comparative assessment.
The choice of reference point also plays a crucial role in modeling behavior. We therefore compare two approaches, using a fixed reference point r^{max} and a stochastic reference point r^{stoc} defined in Section 6.3.3, to assess their impact on stakeholder decisions. For clarity of analysis, we assume that all the potential new members in $\mathcal{M}$ and the REC follow the same reference point selection method (homogeneity of selection approach).

Finally, Table 6.18 presents a summary of utility and PT functions parameterizations in different cases: perfectly rational agents, homogeneously boundedly rational agents, heterogeneously boundedly rational agents, and mixed scenarios with one rational and one boundedly rational agent. This enables in-depth analysis of the combined effects of rationality assumptions and reference point on the strategic and behavioral results of the simulations.

6.7.1. Community new member selection

We compare the SPEs' outcomes of the extensive-form game Γ^2 in Fig. 6.4 for each preference combination, between the parameters set out in the Table 6.18. The game Γ^2 models the NMIP where the community aims to expand by integrating a new member from a list of potential candidates $\overline{\mathcal{M}}$, because users 7 and 11 are eliminated from the initial list $\mathcal{M}$. For each community, we simulated a total of 450 cases, covering all combinations of preference criteria, the possible parameterization scenarios in Table 6.18, and the two types of reference points (fixed and stochastic). We illustrate the most significant combinations of preference criteria in this context.

Num. Scenar.	Users $\mathcal{M}$	REC	PT parameters of users η_a	η_b	η_c	η_d	PT parameters of REC η_a	η_b	η_c	η_d
1	EUT	EUT								
2	PT1	PT1	0.88	0.88	2.25	0.65	0.88	0.88	2.25	0.65
3	PT1	PT2	0.88	0.88	2.25	0.65	0.45	0.45	1.96	0.65
4	PT2	PT1	0.45	0.45	1.96	0.65	0.88	0.88	2.25	0.65
5	PT2	PT2	0.45	0.45	1.96	0.65	0.45	0.45	1.96	0.65
6	PT1	EUT	0.88	0.88	2.25	0.65				
7	PT2	EUT	0.45	0.45	1.96	0.65				
8	EUT	PT1					0.88	0.88	2.25	0.65
9	EUT	PT2					0.45	0.45	1.96	0.65

Table 6.18.: Scenarios of subjective value functions and probability weighting functions parameters for candidates and RECs.

REC in deficit

We begin with the REC in an annual energy deficit situation (Tab. 6.2). It is worth pointing out that, for many combinations of preference criteria, the application of prospect theory increases the number of resulting subgame perfect equilibria, whatever the nature of the reference point used (fixed or stochastic).

We present the actions and outcomes for the two stakeholders maximizing NPV1 (6.14) in Table 6.19. The first row corresponds to the results obtained under the perfect rationality hypothesis, already studied above. For cases with bounded rationality, decisions are made on the basis of the global value specific to PT (6.25). However, to enable direct comparison with the rational case, we express these results in terms of expected utility (6.26), in the table. This allows us to better observe and analyze the differences induced by the application of PT.

Ref. point	Num. Scenar.	Num. SPE	User	PV (kWp)	Stor.	Local prices	Exp. utility user	Exp. utility REC
\	1	1	2	+0	0	D	-21 071.6€	-134 319€
r^{max}	2,3,6,8,9	1	2	+0	0	D	-21 071.6€	-134 319€
	4,5,7	1		+1			-21 840.9€	-133 300.55€
r^{stoc}	2-9	1	2	+0	0	D	-21 071.6€	-134 319€

Table 6.19.: Actions and outcomes of SPEs from game Γ^2 of the energy-deficit REC, for the two stakeholders maximizing NPV1. Results obtained under PT are also shown in terms of expected utility to enable direct comparison.

The calculations include different parameterizations of PT functions and reference points. With the fixed reference point r^{max}, if boundedly rational user 2 follows the PT2 parameters (enterprise level [143]), then the community requires that he invests in an additional PV panel, which implies better results in terms of expected utility for the REC, but not for the user. We do not note any changes in the other cases.

Table 6.20 shows the actions and results when candidates in $\overline{M}$ minimize their total costs C_{tot} (6.13) and the REC minimizes its CDE (6.18).

Ref. point	Num. Scenar.	Num. SPE	User	PV (kWp)	Stor.	Local prices	Exp. utility user	Exp. utility REC
\	1	1	2	+0	0	C	31 410.6€	123.556 tCO2eq
r^{max}	4,7 5	1 8	2	+2	0	I	30 341.12€	119.912 tCO2eq
	2,6,8 3,9	1 8	2	+0	0	C	31 410.6€	123.556 tCO2eq
r^{stoc}	2,4,6,7 3,5	1 32	2	+2	0	I	30 341.12€	119.912 tCO2eq
	8 9	1 32	2	+0	0	C	31 410.6€	123.556 tCO2eq

Table 6.20.: Actions and outcomes of SPEs from game Γ^2, for candidates minimizing C_{tot} and the energy-deficit REC minimizing its CDE. Results obtained under PT are also shown in terms of expected utility to enable direct comparison.

For the maximum reference point r^{max}, if user 2 is boundedly rational with PT2 parameters (corporation level [143]), we have that REC will require that user 2 invests in two additional PV panels. Which leads to better expected utility compared to the rational case, for both stakeholders. Note that if the REC also follows PT2 (scenario 5), then we get 8 SPEs with the same actions. We have the same deviation with the stochastic reference point, but in all cases where user 2 is irrational (PT1 and PT2)! If, in addition, the REC follows PT2, then we have 32 SPEs. Otherwise, we observe the same actions and outcomes as the rational case, with sometimes more SPEs.

REC in surplus

Apart from the situation where NPV1 is the candidate preference criterion and the CDE is the one for the REC, we do not observe any deviations between the different scenarios and combinations of simulated criteria for the REC in energy surplus (Tab. 6.3). We sometimes note one or two additional SPEs.

The actions and outcomes for this special combination are displayed in Table 6.21.

Ref. point	Num. Scenar.	Num. SPE	User	PV (kWp)	Stor.	Local prices	Value user	Value REC
\	1	1	2	+0	+1	C	-21 553.8€	68.786 tCO2eq
r^{max}	2-7	1	∅	\	\	\	\	69.016 tCO2eq
	8,9	1	2	+0	+1	C	-21 553.8€	68.786 tCO2eq
r^{stoc}	2,4,6,7,8 3,5,9	1 2	2	+0	+1	C	-21 553.8€	68.786 tCO2eq

Table 6.21.: Actions and outcomes of SPEs from game Γ^2, for candidates maximizing NPV1 and the energy-surplus REC minimizing its CDE. Results obtained under PT are also shown in terms of expected utility to enable direct comparison.

For the fixed reference points r^{max} case, if the candidates are bounded rationally (PT1 or PT2), then the community chooses no one and stays with its original composition. This is actually a better situation for user 2, as the investment option required by the REC in the rational case (1 in Tab. 6.21) will degrade her expected utility of NPV1. There will be no change if the reference points are stochastic, except that we have 2 SPEs in the case where the community follows the PT2 parameters.

Observations summary

We summarize the conclusions of this subsection.

In the context of the extensive game Γ^2 in Fig. 6.4:

1. Stakeholders' behavior and decisions may differ between cases of perfect rationality and those subject to bounded rationality via PT.
2. The choice parameters is a fundamental element of PT to capture the behaviors. It seems crucial to use a parameterization suited to the type of user or entity under study, whether individuals, organizations or energy communities.
3. The selection of the reference point is also decisive.
4. The combinations of preferences can lead to differences in actions and results for the community and users.

These deviations emphasize the necessity of considering more nuanced behavioral models. Further, they also call for experimental studies with energy communities to estimate suitable parameters for this new mechanism, and thus refine the relevance of behavioral models applied to these specific contexts.

6.7.2. Impact of the decisions order with PT framework

We now consider the point of view of a user $j \in \overline{M}$ by comparing the actions and results of the SPEs for various system configurations in Table 6.18. In addition, we analyze the impact of decision order in this context, by examining the differences between the extensive games Γ^2 in Fig. 6.4 (REC moves first) and Γ_j^1 in Fig. 6.3 (user j moves first) for each preference combination. For each community, we simulated a total of 3600 cases, covering all candidates in $\overline{\mathcal{M}}$, combinations of preference criteria, parameterization scenarios in Table 6.18, and the two types of reference points (fixed and stochastic). Once again, we illustrate the most significant users and combinations.

REC in deficit

In Section 6.7.1, we found that the REC in annual energy deficit (Tab. 6.2) still selects user 2 (Tab. 6.5) as a new member. Two configurations emerged as particularly interesting: 1) stakeholders seek to maximize NPV1 (6.14); and 2) users minimize their total costs (6.13) while the community aims to reduce its CDE (6.18). We focus on the study of user 2's actions and outcomes in these two configurations, for both extensive-form games.

We begin with the homogeneous combination NPV1. In fact, no change is observed in the games Γ_2^1: all parameterization scenarios and the two reference points always lead to the same strategies as in the rational situation. Thus, user 2's strategies remain identical in both games Γ_2^1 and Γ^2, with the exception of the case where the reference point is fixed and user 2 follows the PT2 parameters (see Table 6.19). In this specific configuration, the investment option required by the REC turns out to be unfavorable for user 2, who would benefit more by choosing to remain outside the community.

We examine the second situation, where user 2 minimizes her total cost (C_{tot}), while the community optimizes its CDE. The actions and outcomes at SPEs of the game Γ_2^1 are provided by Table 6.22. The outcomes obtained under the perfect rationality assumption are displayed in the first row. As a reminder, the decision-making process is based on the global value (6.25) for the bounded rationality hypothesis. Nevertheless, we express the outcomes in terms of expected utility (6.26) in the table, in order to enable direct comparison with the perfectly rational case.
We note that user 2's strategy in the rational case differs between the two extensive games (see the first line in Tables 6.20 and 6.22). In the game Γ_2^1, when the reference points are fixed and user 2's parameters follow PT2 (organization level [143]), the results diverge from those obtained in the rational framework.

Ref. point	Num. Scenar.	Num. SPE	In REC	PV (kWp)	Stor.	Local prices	Exp. utility user 2	Exp. utility REC
\	1	1	1	+0	+1	D	30 298.4€	126.527 tCO2eq
r^{max}	4,5,7	1	1	+2	0	I	30 341.12€	119.912 tCO2eq
	2,3,6,8,9	1		+0	+1	D	30 298.4€	126.527 tCO2eq
r^{stoc}	2,4,6,7 3,5	1 4	1	+2	0	I	30 341.12€	119.912 tCO2eq
	8 9	1 4	1	+0	+1	D	30 298.4€	126.527 tCO2eq

Table 6.22.: Actions and outcomes of SPEs from game Γ_2^1, for user 2 minimizing C_{tot} and the energy-deficit REC minimizing its CDE. Results obtained under PT are also shown in terms of expected utility to enable direct comparison.

In this configuration, although the REC's expected utility is improved, that of user 2 is lower. Despite this, user 2 still chooses to join the community. These results are repeated in Γ^2 for the same parameterization scenarios (Tab. 6.20). In other cases, the strategies adopted vary between the two games. If the reference point is stochastic, the observations remain similar for a non-rational user 2 (PT1 or PT2). In all cases, the community would benefit from selecting user 2.

REC in surplus

We are interested in the community with an annual energy surplus (Tab. 6.3), with a priority focus on users 1 and 2.

Profile 1. Table 6.23 presented the actions and outcomes at the SPEs of the game Γ_1^1, where user 1 maximizes the NPV2 and the REC optimizes its CDE. For the fixed reference point r^{max}, we observe differences with the rational scenario when the bounded rationality of the community is expressed with PT2 (organization-level [143]). In this case, if user 1 is irrational with PT1 (individual-level [81]) then he invests in 9 panels, whereas if this user follows PT2 or is considered rational, he chooses 10 panels. We now discuss the stochastic reference point r^{stoc}. If the boundedly rational REC is based on the PT1 parameters (individual-level [81]), then it will lower its local tariffs λ_{iloc} and λ_{eloc}, resulting in an increase of the user's expected value of NPV2. In fact, user 1's strategies are identical in both games Γ^2 and Γ_1^1. However, the community has no interest in selecting candidate 1.

Profile 2. In Section 6.7.1, we found an interesting combination for user 2. In

Ref. point	Num. Scenar.	Num. SPE	In REC	PV (kWp)	Stor.	Local prices	Exp. utility user 1	Exp. utility REC
\	1	1	1	+8	+1	C	-96 849.48€	69.183 tCO2eq
r^{max}	2,4,6,7,8	1	1	+8	+1	C	-96 849.48€	69.183 tCO2eq
	3			+9		C	-96 802.4€	69.183 tCO2eq
	5,9			+10		D	-96 899.35€	69.183 tCO2eq
r^{stoc}	3,5,6,7,9	1	1	+8	+1	C	-96 849.48€	69.183 tCO2eq
	2,4,8					D	-96 627.98	69.183 tCO2eq

Table 6.23.: Actions and outcomes of SPEs from game Γ_1^1, for user 1 maximizing NPV2 and the energy-surplus REC minimizing its CDE. Results obtained under PT are also shown in terms of expected utility to enable direct comparison.

this situation, user 2 optimizes the NPV1 and the REC minimizes its CDE. We compare results from the game Γ^2 at Table 6.21 with those of the game Γ_2^1. For both reference point selections and each parameterization scenario, candidate 2 chooses to enter the REC without investment, which is not advantageous for the community. Then, SPEs of the extensive-form games Γ^2 and Γ_2^1 are strictly different with the stochastic reference point r^{stoc}. In the case of the reference point set at the initial maximum value r^{max}, we have that if the candidates are irrational in Tab. 6.21, then user 2 will not invest, so we have the same strategy as in the game Γ_2^1.

Profile 10. We observed an intriguing combination for user 10 (Tab. 6.5). Table 6.24 shows the actions and outcomes at the SPEs of the game Γ_{10}^1, where user 10 minimizes her total cost while the REC optimizes its NPV1.

Ref. point	Num. Scenar.	Num. SPE	In REC	PV (kWp)	Stor.	Local prices	Exp. utility user 10	Exp. utility REC
\	1	1	0	+3	+1	\	36 464.54€	-43 524.35€
r^{max}	2,3,6,8,9	1	0	+3	+1	\	36 464.54€	-43 524.35€
	4,5,7	2	0	+3	+1	\	36 464.54€	-43 524.35€
			1	+3	+1	D	36 465.09€	-43 441.1€
r^{stoc}	2,3,6,8,9	1	0	+3	+1	\	36 464.54€	-43 524.35€
	4,5,7		1			D	36 465.09€	-43 441.1€

Table 6.24.: Actions and outcomes of SPEs from game Γ_{10}^1, for user 10 minimizing C_{tot} and the energy-surplus REC maximizing its NPV1. Results obtained under PT are also shown in terms of expected utility to enable direct comparison.

In any case, user 10 is always well advised to invest in 3 additional solar panels and a battery. In the majority of parameterization scenarios, user 10 is more inclined to remain independent. An exception occurs when candidates adopt the PT2 parameters (enterprise-level [143]). In this setting, user 10 is either indifferent between remaining alone or joining the REC if r^{max}, or he chooses to integrate the REC if r^{stoc}. Note that the difference between the expected values for the user is 0.554€, which could be negligible. In the game Γ^2 framework, the community always benefits from bringing in this user with this investment profile.

Observations summary

This subsection measured the impact of the order of decision-making in the NMIP on SPEs and behaviors in different stakeholder bounded rationality parameterization scenarios (Tab. 6.18). We summarize our observations.

1. In NMIP, the order of decisions influences the SPEs and behavior of stakeholders even under prospect theory assumptions.

In the context of the extensive game Γ^1 in Fig. 6.3 for any user $j \in \mathcal{M}$:

2. Stakeholders' behavior and decisions may differ between cases of perfect rationality and those subject to bounded rationality via the prospect theory.
3. The reference point selection and an appropriate parameterization scenario for the type of entity studied, are decisive elements in the behavioral decision-making process.
4. The combinations of preference criteria can also influence the results.

6.8. Conclusion

This chapter proposed an original approach to a rather unexplored topic: the integration of a new member into an existing renewable energy community (REC). The New Member Integration Problem (NMIP) is defined for a collaborative community built on a demand-side management (DSM) scheme, allowing the valuation of excess generation in the REC pool and on the retail markets (design D2 in Chapter 4). This problem addresses strategic decisions with both long-term implications, such as investment choices or local price adjustments; and short-term ones, through daily operational decisions linked to energy and financial flow management. In addition, long-term uncertainties concerning the evolution of retail import tariffs have been integrated through three scenarios in the NMIP. We use extensive-form game theory to model the different time horizons of the decision-making process. In particular, short-term decisions of the day-ahead energy scheduling, are modeled using the generalized Nash equilibrium problem studied in Chapter 4. To capture the complex dynamics of the process, two distinct extensive games have been established, each reflecting a different sequence in the order of decision-making. This enabled analysis of the influence of the decision order on the strategies and subgame perfect equilibria. In the first case (Γ^1 in Fig. 6.3), an external user is interested in joining a REC, therefore modeling a situation where the user acts with limited knowledge of the community's reactions. In the second case (Γ^2 in Fig. 6.4), the community is the instigator of its own expansion and selects a new member among a candidates set $\mathcal{M}$.

One of the strengths of our contribution lies in the flexibility and manageability of the proposed models. The theoretical formulation can be extended to encompass a variety of scenarios and stakeholder preference criteria, for instance the minimization of the total cost, carbon emissions or the kWh price; and the maximization of the net present value or the return of investment. Furthermore, it can incorporate both perfect and bounded rationality hypotheses, allowing variation in the parameters of the prospect theory as well as in the selection method of the reference point. This adaptability paves the way for potential extensions, which are discussed in the continuation of the section and in Chapter 7.

We applied our models to a detailed case study, setting three types of renewable energy communities: one in energy deficit, one in surplus, and one close to energy balance on an annual basis. Each community was studied through a set of simulations, including 11 potential candidates willing to join the REC and 25 combinations of preference criteria for both extensive games. We first analyzed the results of heuristic methods proposed by Mustika et al. in [20], and compared them with the SPEs obtained from the game Γ^2 (community moves first). For each candidate preference criterion, if the REC prioritizes financial objectives (e.g., net value or total cost), the heuristic methods can effectively predict the next member selected by the community. However, when the community focuses on carbon emissions or the price per kWh, they may fail to reliably anticipate the REC's decision. In these cases, choices are influenced by the specific characteristics of the SPEs and the preferences of the candidates.

We then explored the impact of the order of decisions, comparing the outcomes obtained from SPEs of the games Γ^1 and Γ^2. The simulations reveal that the sequence of decisions affects the SPEs and the stakeholders' behavior. Indeed, the agents anticipate the reactions of the other participants differently, which influence the strategies adopted. In particular, game Γ^2 (where the community decides first) favors decisions that are more consistent with the objective of the whole community, sometimes to the detriment of the new member, while game Γ^1_j (where user j moves first) reflects the individual interests of user j and thus perhaps to the community's disadvantage. In addition, the nature of preferences, their parameters and their combinations directly influence the NMIP's actions and results, highlighting the significance of strategic choices for all stakeholders.

Finally, we investigated the long-term consequences of decision-making process under risk with the bounded rationality assumption. To this end, prospect theory was used to capture elements of bounded rationality in the models and assess their potential impacts. We tested two sets of parameters for prospect theory, namely at the individual-level and the corporate-level, enabling us to investigate nine different parameterization scenarios. We also compared two reference point selection methods in order to assess the impact of the reference point dependency. Simulations results show that non-rational behavior of stakeholders can generate deviations from the rational cases. These deviations seem particularly sensitive to the reference point selection used and the parameters category chosen for the prospect theory. For example, specific configurations can lead to situations where some users are incentivized to join the community, even though their expected utility is lower. This highlights the importance of properly calibrating model parameters to reflect actual stakeholders' behavior. In conclusion, this analysis suggests the importance of taking into account the individual objectives and subjective perceptions of each stakeholder, whether community or candidates, to better reflect the underlying strategic dynamics and achieve a decision-making process that represents actual behavior under risk.

In this chapter, we have considered that investments are individual, so a natural extension would be to address the case where investments are made jointly. Similarly, we could study the problem of a member leaving a community, for both types of investment. We have assumed finite number of actions available at each LT node, for example, a finite range of investment profiles. Therefore, another extension of our models lies in broadening the set of possible actions. An extension to larger, continuous or infinite action sets, and more complex probabilistic distributions would enable to explore a wider spectrum of possible behaviors and better represent real uncertainties. Nonetheless, there are already a number of opportunities for further research with the models established here. Although our simulations have focused on a specific case study, the proposed framework is flexible enough to handle a wide variety of scenarios. For instance, varied preference criteria, alternative scenarios of retail import price evolution and exogenous uncertainties, other parameters for the prospect theory could be tested, as well as different reference point selections. A logical development would be to diversify preferences within the set of potential new members $\mathcal{M}$, where so far we have considered homogeneity. In addition, we have assumed that the REC and all candidates adopt the same reference point setting, but this element is a subjective notion specific to each individual or entity. An interesting direction would be to examine situations in which this assumption of uniformity is lifted, thereby expanding consideration of the heterogeneity of perceptions and decisions. Despite deploying prospect theory to model decisions in this context, an empirical study applied to energy communities would be required to validate the applicability of this theory and estimate parameter sets suitable for the community mechanism. This empirical approach could offer a more operational insight for actors in the sector. Lastly, from a regulatory point of view and in terms of energy community governance, a key question remains to be clarified: how to define the preferences of an energy community composed of consumers and prosumers with heterogeneous targets, perceptions and priorities? As such, our approach has considered the existing community as a single entity with well-defined collective objectives for long-term decision-making. An in-depth exploration could provide interesting perspectives both for theoretical modeling and for the development of adapted regulations.

CHAPTER 7.

Conclusions and Perspectives

This final chapter summarizes the main contributions of the thesis and provides some perspectives for further research.

7.1. Conclusions

This thesis addresses different problems in the context of renewable energy communities modeling and implementation, through convex optimization and noncooperative game-theoretical frameworks. In this respect, it covers a wide range of interdisciplinary fields, including energy markets, mathematics, operations research and economics. Thus, we proposed specific chapters dedicated to contextualization (Chapter 2) and presentations of fundamental concepts and theoretical results used in this report (Chapters 3 and 5). We first explored the optimal day-ahead scheduling of energy exchanges and members' appliances within RECs using (generalized) Nash equilibrium problems in Chapter 4. We then exploited this approach to develop a theoretical model for the integration of a new member into an existing energy community, presented in Chapter 6. Chapters 4 and 6 are summarized below, providing an overview of the proposed research contributions and findings.

Above all, it should be noted that rather than proposing tools that can be directly used as such, this thesis is rather part of a wider approach aimed at the development and detailed analysis of theoretical models. When possible, our models are backed by numerical results; further, they are flexible enough to be adapted to other configurations or parameters.

In **Chapter 4**, we compared two market designs for the day-ahead energy resources scheduling problems in RECs. The two models, D1 (collaborative management scheme) and D2 (valuing local green surplus production in community and the retail markets), are analyzed under two grid tariff structures T1 (quadratic costs) and T2 (linear costs). Member costs are allocated according to four billing methods. These designs have been formulated as centralized optimization problems and as noncooperative normal-form games, with an analytical and empirical study. The results show that D2 model valuing excess generation, improves REC performance in terms of costs, self-consumption and peak-to-average ratio, with additional savings of 8.36% compared to D1, which already saved 30.14% compared to the case without community. We obtained several theoretical results, such as theorems of equilibria existence, formulations equivalence, efficiency (through the price of anarchy and price of stability) and convergence results that are summarized in Table 4.6 on page 113. We showed that the community total bill obtained with the centralized and decentralized approaches are equivalent (or exhibit a negligible difference, maximum to 1.49%) for the studied cost allocation methods. Similarly, we showed that the individuals' bills obtained ex-post from the faster centralized formulations and from the decentralized models are similar in most studied configurations, except for the continuous billing allocation with the T1 grid tariff. So, this last setting required a solution based on variational inequality. We also studied the impact of retail electricity prices and the two grid tariff structures on the operation of a REC with design D2. We revealed that there is a threshold in the retail import price, a function of the difference between community and retail import/export prices, beyond which the economic gains of operating as a REC increase significantly, for both tariff structures. Thus, REC mechanisms can offer partial protection against price volatility, but it is crucial to define an appropriate network cost structure and cost-sharing method.

The **Chapter 6** covers a fairly new topic: the integration of a new member into an existing renewable energy community (NMIP). The problem has been modeled through extensive-form game theory to reflect strategic decisions taken at the initial time (t=0), which will have long-term consequences (investments, local price adjustments), as well as short-term operational decisions (day-ahead scheduling). Uncertainty related to the retail import price's evolution over the complete horizon, was also included via three distinct scenarios. Two decision-making sequences were studied: (1) an external end-user suggests joining the REC, and (2) the community selects a new member from a set of potential candidates. The theoretical models developed in this chapter, offer a high degree of flexibility and manageability, making it possible to integrate different long-term preference criteria (total cost, carbon emissions, price of the kWh, etc.) and rationality hypotheses, ranging from perfect rationality to bounded rationality leverage by prospect theory (PT). A case study with three types of RECs (energy deficit, surplus and equilibrium) was applied to our models. Each REC was studied through a series of simulations, including 11 potential candidates and 25 combinations of preference criteria. An analysis was carried out on the results of heuristic methods from the literature [20], designed to rank potential new users according to their fit with the existing community. Compared to the subgame perfect equilibria (SPEs) obtained when the community initiates integration, these metrics can effectively predict the selected profile when the REC has financial criteria such as cost reduction, etc. However, their reliability decreases if the REC follows criteria such as carbon emissions or price per kWh. Simulations show that the order of decisions has a significant influence on SPEs and stakeholders behavior. When the REC decides first, choices promote the collective objective, sometimes to the detriment of the potential new member. Conversely, when users act first, decisions reflect their individual interests, sometimes to the disadvantage of the REC. Given bounded rationality via prospect theory, we noted that the non-rational behavior of stakeholders can generate significant deviations from rational cases. These deviations depend on the type of parameters used for PT, i.e., whether they are based on the individual or organizational level; and on the reference point selection method. In summary, this analysis illustrates the necessity of considering individual preferences and subjective perceptions of each stakeholder, whether candidate or REC, into the models, in order to better capture the underlying strategic dynamics and ensure a decision-making process that reflects actual behavior under risk.

7.2. Perspectives

Several directions for further research on an extended framework could be investigated.

7.2.1. Chapter 4: day-ahead scheduling - outlook

The Chapter 4 has explored the day-ahead scheduling inside renewable energy communities. It is essential to examine the limitations of the approaches adopted and to identify research and application perspectives to extend and refine this work.

Technical considerations. Throughout this work, simplified designs have been assumed to represent technical constraints. These models need to be adapted to incorporate more operational details. The inclusion of energy losses in storage would add a layer of realism, although they should not alter the theoretical results demonstrated in the Chapter 4. Some appliances can have a working duty cycle (e.g., dishwasher) that generally cannot be stopped and restarted at any time. Managing such loads often requires considering binary variables, and therefore representing the demand-side management model as a Mixed-Integer Linear Programming (MILP) Problem, see e.g., [28]. A significant extension of our model would be to integrate the physical constraints associated with the network in our analysis. It would then be relevant to take into account the Optimal Power Flow (OPF) equations adapted to LV radial networks. To overcome the non-convex nature of the original equations, a possible approach is to recast the original OPF problem into a Second Order Cone Programming (SOCP) formulation. This relaxation offers an effective compromise, considering power losses while maintaining good computational performance [171, 76]. With this in mind, other grid costs structures could be studied.

Real-time deviations from the optimal schedule. The work in Chapter 4 schedules in day-ahead the optimal energy exchanges and usage of energy assets. However, in real time, deviations with respect to the optimal day-ahead schedules may occur, due to (1) errors in the forecast of renewable generation and to the consumption of nonflexible appliances (exogenous factor), and (2) the actual behavior of the members, who may not necessarily follow the recommendations (by mistake, by lack of interest, etc.) even if the possible existence of game-theoretical equilibria should guarantee their adhesion to the solution (endogenous factor). Such deviations may have a significant impact on the whole community welfare, and must thereby be anticipated.

1. In this thesis, we adopted a deterministic point of view. Uncertainty would make the analysis established here more complex, but there are possible frameworks to study these aspects. The day-ahead model could be formulated as a stochastic generalized Nash equilibrium problem (SGNEP) [172], a robust game [173], or a stochastic game [86, 17].
2. We could design appropriate penalty mechanisms to deal with optimal schedule deviations, by e.g., charging the "faulty" member with the cost of desoptimization due to his deviation, or impacting her electricity supply, etc. These mechanisms should be aligned with existing regulatory policy guidelines. A study of regulations on the management of energy communities and an assessment of the social acceptability of penalties, would be essential to ensure their effective and equitable implementation.

Heterogeneity of objectives within the community. Apart from data privacy issues, the use of decentralized approaches for modeling communities remains of practical significance if community members pursue individual objectives of different natures (e.g., bill minimization for member 1, CO2 emissions minimization for member 2, etc.). This kind of diversity in members' objectives could complicate optimal coordination within the REC, but could also lead to results that are more tailored to each individual's specific needs. This would increase the interest of end-users in getting involved in energy communities. It would, therefore, be useful to develop theoretical results that take this heterogeneity of objectives into account.

Bounded rationality and risk attitudes. In Chapter 4, we have assumed that REC's members are rational economic agents. However, as Kahneman and Tversky show, individuals are not actually rational and the expected utility theory (the basis of game theory, [8]) cannot predict the actual behavior of decision-makers under risk [77, 81]. To ensure active participation from end-users, it would be relevant to incorporate the bounded rationality into our models. In this regard, prospect theory [77, 81], which provides a model of real decision-making under risk, could offer an interesting approach. Although we applied it to model a long-term problem in Chapter 6, the application of this theory in this short-term setting presents some challenges. In particular, the parameterization of the members' subjective value and weighting functions, as they do not necessarily address the same attitude to risk, nor the same perception of values, and these can evolve over time! Another reason lies in the non-convexity and non-concavity of the individuals' global value function, which could complicate the analysis compared to the convexity assumed in our theoretical results. However, a recent paper [174] proposes an approach that could provide a basis for overcoming this theoretical limitation.

7.2.2. Chapter 6: NMIP - outlook

Although Chapter 6 proposed an innovative theoretical framework for integrating new members into an existing renewable energy community, certain assumptions and simplifications were necessary to scope the problem. These choices raise several potential directions for further research, and highlight certain limitations that deserve further thorough treatment in future studies.

Natural extension. In Chapter 6, we assumed that investments were made on an individual basis. So, a first natural extension would be to adapt the proposed models considering joint investments [175]. In parallel, another research direction would be to study the opposite problem, i.e., the departure of a member from an energy community, for both investment configurations. Finally, we could also consider that investments are not only made at t=0.

Infinite tree. The models developed in Chapter 6 offer finite branching trees. We have limited the investment options to a finite number and considered a finite probability distribution for the evolution of the retail import tariff. An extension of this work is to consider infinite branches where the user can optimize her investment and consider more complex probability distributions, thanks notably to Stackelberg games. We have also restricted to finite horizons, choosing to treat the problem for a finite number of years. If we want to avoid predefining the duration of a process or collaboration, it becomes necessary to consider an infinite horizon tree. In repeated game theory, a distinction is made between infinitely repeated games and discounted infinitely games [176, 177]. However, in such games, strategies can become exceedingly complicated and overly complex for users to implement. One perspective is that end-users will learn their profiles over the course of repetitions. Therefore, it could be worthwhile to study the existence of equilibria for simpler strategies and their computations [178, 179].

Interaction with other actors. Although this thesis has focused on the internal interactions in RECs, it is obvious that they do not operate in isolation. In Chapter 2 we described the multiple actors involved in the electrical power system, such as electricity suppliers, aggregators, DSO, etc. These external actors may then find themselves interacting with energy communities and have their operations and decisions directly impacted. For example, an electricity supplier interacting with day-aheard markets, could adjust the tariffs offered to customers who are members of a REC (e.g., [160, 13, 161]), while a DSO might have to adapt its tariffs or infrastructure to integrate local fluctuations in energy injections and offtakes (e.g., [84]). An interesting research direction would be to model strategic interactions between energy communities and other system actors using Multi-Leader-Follower Stackelberg games [180]. These complex games could be designed to improve knowledge of the impact of RECs on the power system and to formulate recommendations to facilitate their implementation into the regulatory and operational framework. The tree-based formulation proposed in Chapter 6 could serve as an initial foundation for such models.

Cooperative game. In this thesis, we have only mobilized noncooperative game theory to model strategic interactions between members, the REC and external end-users. An interesting approach for the integration of a new member could be to consider the cooperative game framework [7]. Indeed, it would enable to study this problem from the point of view of the formation of new coalitions between candidates and the already established energy community, using concepts such as the core, the Shapley value, coalition stability, etc.

Sociological studies to calibrate prospect theory parameters. A restriction lies in the absence of empirical data to calibrate the parameters of prospect theory in the context of energy communities. These parameters, such as diminishing sensitivity coefficients, the loss aversion coefficient or probability weights, were set in our simulations using standard values from the literature [81, 143, 150]. However, their relevance to energy communities remains uncertain. Sociological and behavioral studies to estimate these parameters contextually would better represent stakeholders' actual behavior, improve model consistency and so ensure that results reflect not only abstract theories, but also the social and psychological dynamics observed in energy communities.

APPENDIX A.

Nash Equilibrium Problem

A.1. Proof Theorem 4.2 and 4.3

We prove the two theorems of Chapter 4 by relying on specific properties of our Nash equilibrium problem (4.25), and by resorting to results from Potential Game theory and Variational Inequality theory presented in Chapter 3.

Let $G = (\mathcal{N}, \Omega, (b_i)_{i=1}^N)$ such as $\Omega := \prod_{i\in\mathcal{N}} \Omega_i$, be the NEP in (4.25) with tariff T1. It can first easily be shown that the following properties hold:

P1. *Each b_i is continuously differentiable on Ω.*
P2. *Each Ω_i is closed and convex.*
P3. *Each Ω_i is bounded.*
P4. *For any Θ_{-i}, $b_i(\cdot, \Theta_{-i})$ is convex on Ω_i.*

We can exhibit that NEP G in (4.25) is a Potential Game for the four cost distribution methods described in Section 4.3.2. Specifically, the NEPs with [EB,NET,VCG] are weighted potential games (WPGs), with the potential function $P = f^{\mathrm{D1}}$ in (4.17) and $\omega = (K_i)_{i\in\mathcal{N}}$. In other words, by the Definition 3.27, for all $i \in \mathcal{N}$, given $\Theta_{-i} \in \Omega_{-i}$,

$$b_i(\Theta_i, \Theta_{-i}) - b_i(\Theta'_i, \Theta_{-i}) = K_i(f^{\mathrm{D1}}(\Theta_i, \Theta_{-i}) - f^{\mathrm{D1}}(\Theta'_i, \Theta_{-i})), \quad \forall \Theta_i, \Theta'_i \in \Omega_i.$$

Note that [EB] can also be written as a PG, since for all $i \in \mathcal{N}$, $K_i = 1/N$. The NEP G with [CB] is an exact potential game, with the potential function defined as:

$$P(\Theta) = f^{\mathrm{D1}}(\Theta) - \frac{\alpha}{2} \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{N}} l_i^t . L_{-i}^t, \quad \forall \Theta \in \Omega.$$

For the remainder of this work, we name a potential game any game with a weighted or exact potential function.

Given properties **P2** and **P3**, we have that the feasible set of each player is compact, moreover the potential functions are continuous, hence we assert that the NE set of G is nonempty by applying Theorem 3.11, proving the first point of Theorem 4.2.

It can easily be shown by definitions of potential game and Nash equilibrium (4.26), that an optimal solution for the centralized optimization problems (4.19) and (4.31) is a NE of the game G with [EB,NET, VCG] and [CB] respectively. Conversely, as the potential functions are continuously differentiable and convex on Ω, a NE is an optimal solution of the problem (4.19) for [EB, NET, VCG] or problem (4.31) for [CB], by Theorem 3.10. In conclusion the characteristics of our NEPs can be obtained by solving standard optimization problems. In the case of the daily distribution methods, we have proved that the set of Nash equilibria coincides with the optimal solutions' set of the centralized problem (4.19), i.e.,

$$\mathrm{NE}(G) = X_{\mathrm{opt}}(P_1). \tag{A.1}$$

In addition, each member's individual bill is given by the total energy costs (=social cost) times a strictly positive constant K_i. Besides, for all the cost distribution methods (even [CB]), we have for any profile $\Theta \in \Omega$ that $\mathrm{SC}(\Theta) = f^{\mathrm{D1}}(\Theta)$. Therefore, for each member, all the NEs lead to the same payoff function value under [EB,NET,VCG]:

$$\forall \Theta^*, \Theta' \in \mathrm{NE}(G), \forall i \in \mathcal{N},\ b_i(\Theta^*) = b_i(\Theta').$$

Even if there are multiple NEs, we can state that the individual bill for each player is constant over the NE set of the game G, showing Theorem 4.3. From this, we can deduce for [EB,NET,VCG] that:

$$\mathrm{PoA}(G) := \frac{\max_{\Theta^* \in \mathrm{NE}(G)} \mathrm{SC}(\Theta^*)}{\min_{\Theta' \in \Omega} \mathrm{SC}(\Theta')} = \frac{\max_{\Theta^* \in \mathrm{NE}(G)} f^{\mathrm{D1}}(\Theta^*)}{\min_{\Theta' \in \Omega} f^{\mathrm{D1}}(\Theta')} \underset{(\mathrm{A.1})}{=} 1.$$

In conclusion, all Nash equilibria are efficient.

We further resort to the theory of finite-dimensional Variational Inequalities (VIs), in order to prove the second point of Theorem 4.2.
For each player $i \in \mathcal{N}$, the strategy set Θ_i verifies **P2** and is assumed nonempty. The payoff functions b_i verify **P1** and **P4**. Hence, the NEP G is equivalent to the VI problem $\mathrm{VI}(\Omega, F)$, with $F(\Theta) := (\nabla_{\Theta_i} b_i(\Theta))_{i=1}^N$, by Proposition 3.1. In other words, the set of Nash equilibria coincides with the variational solutions,

i.e., $\mathrm{NE}(G) = \mathrm{SOL}(\Omega, F)$. Since each Ω_i verifies **P3**, hence so is Ω. Thus, by Theorem 3.5, the Theorem 4.2 is also established.

A.2. PDA Convergence

The Proximal Decomposition Algorithm (PDA) is described by Algorithm 2 in Section 3.4 of Chapter 3. As a reminder, instead of a single NEP, we solve a sequence of strongly convex sub-problems with a particular structure which are guaranteed to converge under some technical conditions.

We consider a regularization of the original $\mathrm{VI}(\Omega, F)$, given by $\mathrm{VI}(\Omega, F + \tau(I - y^k))$, where I is the identity map, y^k is a fixed vector in $\mathbb{R}^n$ at the iteration k, and τ is a positive constant. At each iteration $k+1$, the players update their strategies simultaneously (via a Jacobi scheme) by minimizing their bill while perceiving the recently available value of the aggregate net load. The regularized VI is therefore equivalent to the following regularized game G_{τ,Θ^k} where each player $i \in \mathcal{N}$ solves:

$$\begin{aligned} &\min_{\Theta_i} \; b_i(\Theta_i, \Theta_{-i}^k) + \frac{\tau}{2}||\Theta_i - \Theta_i^k||^2 \\ &\text{s.t. } \; \Theta_i \in \Omega_i. \end{aligned} \tag{A.2}$$

If the regularization parameter $\tau > 0$ is sufficiently large, then the game G_{τ,Θ^k} is a strongly monotone NEP with a unique equilibrium that can be computed by the best-response algorithm. The connection between the solution of the regularized game and our NEP G is given by Proposition 3.4.

Convergence of PDA is studied in Theorem A.1.

Theorem A.1. *Let $G = (\mathcal{N}, \Omega, (b_i)_{i=1}^N)$ be a NEP as in (4.25), with tariff T1 and a nonempty set solution. If*

1. *the regularization parameter τ satisfies for [EB,NET,VCG]:*

$$\tau > 4\alpha(N-1)\max_{i\in\mathcal{N}} K_i \tag{A.3a}$$

and for [CB]:

$$\tau > 2\alpha(N-1) \tag{A.3b}$$

2. *a ρ is chosen such that $\rho \subset [R_m, R_M]$, with $0 < R_m < R_M < 2$,*

then, any sequence $\{\Theta^k\}_{k=1}^{\infty}$ generated by PDA converges to a Nash equilibrium *of the game G.*

This section describes how the sequence generated by the proximal decomposition algorithm converges to a solution of the game G for [CB]. To do this, we exploit the equivalence between NEPs and VIs, following similar arguments than in [11, 122].

Let $G = (\mathcal{N}, \Omega, (b_i^{\mathrm{CB1}})_{i=1}^N)$ be a NEP (4.25), with tariff T1 and the continuous cost allocation method (4.24a). We showed in the previous section A.1 that the NE set of the game is non-empty, compact and coincides with the solution set of the VI problem VI(Ω, F), with $F(\Theta) = (\nabla_{\Theta_i} b_i^{\mathrm{CB1}}(\Theta))_{i=1}^N$. By Theorem 3.14, we need to show that: (a) the mapping F is monotone on Ω; (b) the regularization parameter τ is large enough such that the $N \times N$ matrix $\Upsilon_{F,\tau} = \Upsilon_F + \tau I_N$ is a P-matrix (i.e., all principal minors are positive). The matrix $\Upsilon_{F,\tau}$ is related to the regularized VI$(\Omega, F + \tau.(I - y^k))$, with Υ_F defined by

$$[\Upsilon_F]_{ij} := \begin{cases} \upsilon_i^{\min} & \text{if } i = j \\ -\upsilon_{ij}^{\max} & \text{if } i \neq j \end{cases} \tag{A.4}$$

$$\upsilon_i^{\min} := \min_{\Theta \in \Omega} \lambda_{\text{least}}(J_i F_i(\Theta)) \tag{A.5}$$

$$\upsilon_{ij}^{\max} := \max_{\Theta \in \Omega} ||J_j F_i(\Theta)|| \tag{A.6}$$

where $J_i F_i(\Theta)$ and $J_j F_i(\Theta)$ are partial Jacobian matrices of F, and $\lambda_{\text{least}}(.)$ is the smallest eigenvalue of the symmetric part[1] of the argument matrix. We say that $J_j F_i(\Theta)$ is the Jacobian of $F_i(\Theta)$ with respect to Θ_j.

Proof of (a). According to Definition 3.23, a mapping function $F : \Omega \to \mathbb{R}^n$ with Ω closed and convex, is monotone on Ω when

$$(\Theta - \Theta')^\top (F(\Theta) - F(\Theta')) \geqslant 0, \quad \forall \Theta, \Theta' \in \Omega.$$

In the [CB] setting, F is C^1 and its Jacobian matrix JF is symmetric, which allows us to assert that F is monotone if its symmetric Jacobian is a positive semidefinite matrix by Proposition A.1 [122].

Proposition A.1 ([99])**.** *Let $F : \mathcal{Q} \subseteq \mathbb{R}^n \to \mathbb{R}^n$ be C^1 on the open convex set $\mathcal{U}$. The following statements are valid.*

- *F is monotone on $\mathcal{U}$ if and only if $JF(x)$ is positive semidefinite[a] for all $x \in \mathcal{U}$,*

[1]The symmetric part of a matrix A is $\frac{1}{2}(A + A^\top)$. So, A is symmetric if and only if $A = \frac{1}{2}(A + A^\top)$.

- *F is strictly monotone on $\mathcal{U}$ if and only if $JF(x)$ is positive definite*[b] *for all $x \in \mathcal{U}$,*

[a] A symmetric matrix $A \in \mathrm{M}_n(\mathbb{R})$ is said to be positive-semidefinite if: $\forall x \in \mathbb{R}^n \setminus \{0\}, x^\top Mx \geqslant 0$.
[b] If the above inequality is strict.

We show that the Jacobian matrix JF is a positive semidefinite matrix. We can write the block elements of $JF(\Theta)$ for all $i, j \in \mathcal{N}$, as

$$J_iF_i(\Theta) := \nabla^2_{\Theta_i,\Theta_i} b_i(\Theta) = \mathrm{Diag}(H_i^1, \dots, H_i^T, \mathbf{0}) \tag{A.7}$$
$$J_jF_i(\Theta) := \nabla^2_{\Theta_j,\Theta_i} b_i(\Theta) = \mathrm{Diag}(J_jF_i^1, \dots, J_jF_i^T, \mathbf{0}) \tag{A.8}$$

where $\mathbf{0}$ denoting zero matrix, H_i^t and $J_jF_i^t$ are defined by

$$H_i^t := \begin{pmatrix} 0 & \cdots & 0 & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 & 0 \\ 0 & \cdots & 0 & 2\alpha & -2\alpha \\ 0 & \cdots & 0 & -2\alpha & 2\alpha \end{pmatrix} \tag{A.9}$$
$$J_jF_i^t := \begin{pmatrix} 0 & \cdots & 0 & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 & 0 \\ 0 & \cdots & 0 & \alpha & -\alpha \\ 0 & \cdots & 0 & -\alpha & \alpha \end{pmatrix}. \tag{A.10}$$

After algebraic manipulations, it follows that the Jacobian matrix is a block diagonal matrix: $JF(\Theta) := \mathrm{Diag}(\mathbf{0}, B^1, \dots, B^T)$ with $B^t := \alpha(D + E)$ for all $t \in \mathcal{T}$ given by

$$D := \begin{pmatrix} 1 & -1 & 0 & \cdots & 0 & 0 \\ -1 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \ddots & & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & -1 \\ 0 & 0 & 0 & \cdots & -1 & 1 \end{pmatrix} \tag{A.11}$$
$$E := ((-1)^{m+v})_{m,v \in \{1,\dots,2N\}}. \tag{A.12}$$

A block diagonal matrix is positive semidefinite if and only if each diagonal block is positive semidefinite. Furthermore, the sum of positive semidefinite matrices is positive semidefinite. Since this is trivial for the null matrix and $\alpha > 0$, if we show that D and E are positive semidefinite, then the proof is complete.

In fact, D is also a block diagonal matrix. The eigenvalues of D are just the union of eigenvalues of each block. As each block is identical, we can easily calculate the eigenvalues 0 and 2 which are both nonnegatives. So, D is positive semidefinite.

The rank of the matrix E is equal to 1, thus its kernel is of dimension $2N-1$ by Rank-nullity theorem. Therefore, 0 is an eigenvalue of multiplicity $2N-1$. We only need to find a vector that is not in the kernel, see for example: $\begin{pmatrix} 1 & -1 & 1 & -1 & \dots & 1 & -1 \end{pmatrix}$, which associated eigenvalue is $2N$. Therefore, the matrix E is positive semidefinite. ■

Proof of (b). We determine the value of τ such as $\Upsilon_{F,\tau}$ is a P-matrix. The matrix (A.7) corresponds to the user i's Hessian matrix. Because b_i is convex on Ω_i (**P4**), its Hessian matrix is positive semidefinite. Therefore, we can state that $\upsilon_i^{\min} = 0$ in (A.5), for all $i \in \mathcal{N}$. It remains to estimate the values of $\upsilon_{ij}^{\max}$ in (A.6) for all $i, j \in \mathcal{N}$, $i \neq j$. Considering $J_j F_i(\Theta) = \text{Diag}(J_j F_i^1, \dots, J_j F_i^T, \mathbf{0})$ and $J_j F_i^t$ as in (A.10), we have: $\upsilon_{ij}^{\max} \leqslant 2\alpha$.

Therefore, $\Upsilon_{F,\tau}$ is a P-matrix if the following condition is satisfied [122, Prop. 7] for all $i \in \mathcal{N}$,

$$\sum_{j \in \mathcal{N} \setminus \{i\}} \left(\frac{\upsilon_{ij}^{\max}}{\upsilon_i^{\min} + \tau} \right) \leqslant \sum_{j \in \mathcal{N} \setminus \{i\}} \frac{2\alpha}{\tau} \leqslant \frac{2\alpha(N-1)}{\tau} < 1. \tag{A.13}$$

Consequently, any parameter τ such as:

$$\tau > 2\alpha(N-1) \tag{A.14}$$

holds the criterion (A.13). This completes the proof. ■

The case of [EB] distribution can be verified by a similar reasoning, with $\tau > 4\alpha(N-1).N^{-1}$.

Remark A.1. The proof is less straightforward in the case of [NET,VCG]. Using similar reasoning, we obtain a large enough regularization parameter τ such that the N-square matrix $\Upsilon_{F,\tau}$ is a P-matrix. For every user $i \in \mathcal{N}$, we have that $\upsilon_i^{\min} = 0$ in (A.5). Besides, we can estimate the values of $\upsilon_{ij}^{\max}$ in (A.6) for all $i, j \in \mathcal{N}$, $i \neq j$:

$$\begin{aligned}\max_{\Theta\in\Omega} ||J_j F_i(\Theta)|| &\leqslant \max_{t\in\mathcal{T}}\Big(\max_{\Theta\in\Omega} \lambda_{\max}(J_j F_i^t)\Big)\\ &\leqslant 4\alpha(N-1)\max_{i\in\mathcal{N}} K_i. \end{aligned} \tag{A.15}$$

where $\lambda_{\max}(.)$ is the largest eigenvalue of the argument matrix. However, the function F is not monotone, but it is a P_0-function[2] on Ω that is convex and compact. Furthermore, the NE set is non-empty and compact by Theorem 4.2 and we noted convergence to a NE in our case-study framework. We refer readers interested in nonmonotone VI to [181].

Another way is based on the particular structure of the game. Since we are dealing with a potential game [116], we know by Theorem 4.3.1 and Theorem 3.9 that solving $G = (\mathcal{N}, \Omega, (b_i)_{i=1}^N)$ amounts to finding all equilibria of the game $G_P = (\mathcal{N}, \Omega, (P)_{i=1}^N)$ with $P = f^{\mathrm{D1}}$ in (4.17). Because $\nabla P = \nabla f^{\mathrm{D1}}$ is monotone, we can obtain a Nash equilibrium via the PDA.

Remark A.2. As the mapping function F is monotone for [EB, CB], we can conclude by Theorem 3.6 that the NE set of the game G is convex.

[2] F is a P_0-function on Ω if for all pairs of distinct tuples $x, y \in \Omega$, an index i exists such that $x_i \neq y_i$ and $(x_i - y_i)^\top(F_i(x) - F_i(y)) \geq 0$. If the inequality is strict, then F is a P-function.

APPENDIX B.

Generalized Nash Equilibrium Problem

B.1. Proof Theorem 4.4

Let $\mathcal{G} = (\mathcal{N}, (\Omega_i)_{i=1}^N, (b_i)_{i=1}^N)$ be the GNEP in (4.27). We recall the notation used in Chapter 4. For each member $i \in \mathcal{N}$, we denote $\overline{\Omega}_i \subseteq \mathbb{R}^{n_i}$ as the player i individual constraints set, which we assume nonempty. We write the shared internal balance constraints (4.13) as $h(\Theta) := (\sum_{i\in\mathcal{N}} e_i^{com,t} - i_i^{com,t})_{t\in\mathcal{T}}$. We have the feasible set of member i given by (4.28)

$$\Omega_i(\Theta_{-i}) := \left\{\Theta_i \in \overline{\Omega}_i \mid h(\Theta_i, \Theta_{-i}) = 0\right\}.$$

It can be shown that the following properties hold for both T1 and T2 pricing:

P1. *Each b_i is continuously differentiable.*
P2. *Each $\overline{\Omega}_i$ and $\Omega_i(\Theta_{-i})$ are closed and convex.*
P3. *Each $\overline{\Omega}_i$ and $\Omega_i(\Theta_{-i})$ are bounded.*
P4. *For any Θ_{-i}, $b_i(\cdot, \Theta_{-i})$ is convex.*
P5. *The function h is continuous and componentwise convex.*

Furthermore, we have the joint strategy set (4.29) reads

$$\mathcal{C} := \left\{\Theta \in \mathbb{R}^n \mid \Theta_i \in \overline{\Omega}_i \ \forall i \in \mathcal{N},\ h(\Theta) = 0\right\},$$

which is closed, convex and where all the shared constraints are linear. Then, the game $\mathcal{G}$ belongs to the jointly convex GNEPs subclass by Definition 3.20 and we have for each user $i \in \mathcal{N}$

$$\Omega_i(\Theta_{-i}) = \{\Theta_i \in \mathbb{R}^{n_i} \mid (\Theta_i, \Theta_{-i}) \in \mathcal{C}\}.$$

Since all the hypotheses have been verified, we can apply Proposition 3.3, which established a link between our GNEP $\mathcal{G}$ and a suitable VI problem. In this way, an equilibrium of the game $\mathcal{G}$ can be calculated by solving the VI problem $\text{VI}(\mathcal{C}, F = (\nabla_{\Theta_i} b_i)_{i=1}^N)$. Proposition 3.3 states that every solution of the $\text{VI}(\mathcal{C}, F)$ is a generalized Nash equilibrium of the GNEP $\mathcal{G}$, i.e., $\text{SOL}(\mathcal{C}, F) \subseteq \text{GNE}(\mathcal{G})$. We do not have, however, that any GNE of $\mathcal{G}$ is also a solution to the associated VI. Solutions of the GNEP that are also solutions of the VI are called variational equilibria (VEs) (see Definition 3.25, [106]). We note $\text{VE}(\mathcal{G})$ the set of VEs of the jointly convex GNEP $\mathcal{G}$.

We then analyze and compute the variational solutions of the original GNEP (4.27) for both tariff T1 and T2. Since the joint strategy set $\mathcal{C}$ holds **P3**, the Theorem 3.5 guarantees the existence of a VE and thus ensure the existence of a GNE for the game (4.27). Theorem 4.4 is therefore established.

B.2. Proof Theorem 4.5

We exploit connections with potential games in the case of GNEPs [118] (see Section 3.3.2). The jointly convex GNEP $\mathcal{G} = (\mathcal{N}, (\Omega_i)_{i=1}^N, (b_i)_{i=1}^N)$ in (4.27), for both tariffs T1 and T2, is

1. a WPG with $P(\Theta) = f^{\text{D2}}(\Theta)$ in (4.18) and $\omega = (K_i)_{i \in \mathcal{N}}$, for [NET,VCG] billings.
2. An EPG with $P(\Theta) = f^{\text{D2}}(\Theta)/N$, for [EB] billing.

An EPG for [CB] with the billing function 4.24b, with

3. In the case of tariff T1

$$P(\Theta) = f^{\text{D2}}(\Theta) - \frac{\alpha}{2}.\sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{N}} l_i^t.L_{-i}^t \tag{B.1}$$

4. In the case of tariff T2, $P(\Theta) = f^{\text{D2}}(\Theta)$.

For the sake of clarity, we pose the problem of minimizing the potential function on $\mathcal{C}$ as

$$\begin{aligned} \min_{\Theta} \; & P(\Theta) \\ \text{s.t. } \; & \Theta \in \mathcal{C}, \end{aligned} \tag{B.2}$$

without distinguishing between the centralized problems (4.20) and (4.32).

By Theorem 3.12, an optimal solution for the centralized optimization problem B.2 is a GNE of the GNEP. As a reminder, we have $\text{SC} = f^{\text{D2}}$. Thus, we have, by definition, that the price of stability is equal to 1, except for [CB] with tariff 1

$$\mathrm{PoS}(\mathcal{G}) = \frac{\min_{\Theta^* \in \mathrm{GNE}(\mathcal{G})} \mathrm{SC}(\Theta^*)}{\min_{\Theta \in \mathcal{C}} \mathrm{SC}(\Theta)} = \frac{\min_{\Theta^* \in \mathrm{GNE}(\mathcal{G})} f^{\mathrm{D2}}(\Theta^*)}{\min_{\Theta \in \mathcal{C}} f^{\mathrm{D2}}(\Theta)} = 1.$$

As mentioned in Remark 3.2, a GNE Θ^* of the GNEP $\mathcal{G}$ (4.27) does not always minimize the potential function over the joint strategy set $\mathcal{C}$ (4.29), due to shared constraints.

Nevertheless, it can be shown that the PGs for [EB,CB] have their variational equilibria set coinciding with the optimal solutions set of B.2. We noted in Appendix B.1, that VEs correspond to GNEs of the game $\mathcal{G}$, which are also solution to the associated $\mathrm{VI}(\mathcal{C}, F = (\nabla_{\Theta_i} b_i)_{i=1}^N)$. In fact, (4.20) and (4.32) are convex optimization problems with continuously differentiable objective functions, hence a point Θ^* is a global minimum if and only if

$$(\Theta' - \Theta^*)^\top \nabla P(\Theta^*) \geqslant 0, \quad \forall \Theta' \in \mathcal{C} \tag{B.3}$$

by the minimum principle 3.10. Because $\nabla P = F$, the equation B.3 is equivalent to

$$(\Theta' - \Theta^*)^\top F(\Theta^*) \geqslant 0, \quad \forall \Theta' \in \mathcal{C}, \tag{B.4}$$

and so $\Theta^* \in \mathrm{SOL}(\mathcal{C}, F)$. Hence, the VEs can be obtained by solving a standard optimization problem. For both [CB] under tariff 2 and [EB], this optimization problem corresponds to the centralized problem (4.20); we have: $X_{\mathrm{opt}}(P_2) = \mathrm{VE}(\mathcal{G}) \subseteq \mathrm{GNE}(\mathcal{G})$. In this way, the price of anarchy restricted to game's variational equilibria is equal to 1:

$$\mathrm{PoA}(\mathcal{G}_{|\mathrm{VE}}) = \frac{\max_{\Theta^* \in \mathrm{VE}(\mathcal{G})} f^{\mathrm{D2}}(\Theta^*)}{\min_{\Theta \in \mathcal{C}} f^{\mathrm{D2}}(\Theta)} = 1.$$

In addition, each member's individual bill is given by the total energy costs (= social cost) divided by the number of members with [EB] cost allocation method. Therefore, for each member, all the VEs lead to the same cost function value under [EB]. This proves Theorem 4.5.

B.3. PDA with shared constraints convergence

We focus on the distributed computation of the GNEs, especially the VEs. Inspired by the VI framework proposed in [124, 182], we apply the PDA with coupling constraints.

Distributed algorithms require the joint strategy set to be a Cartesian product, so that a VE cannot be computed from the current VI formulation $\mathrm{VI}(\mathcal{C}, F = (\nabla_{\Theta_i} b_i)_{i=1}^N)$ associated with the GNEP $\mathcal{G}$ in (4.27). We decouple the members' feasible sets by converting the global constraints into penalty terms in the objective functions. A new player $N+1$ is also introduced. The extended NEP reads

$$\mathcal{G}_{ext} := \begin{cases} \min\limits_{\Theta_i \in \overline{\Omega}_i} \; b_i(\Theta_i, \Theta_{-i}) + \pi^\top h(\Theta) & \forall i \in \mathcal{N} \\ \min\limits_{\pi \in \mathbb{R}^T} \; - \pi^\top h(\Theta) & i = N+1. \end{cases} \tag{B.5}$$

This new player can be considered as a central operator (e.g., a community manager) controlling a price variable $\pi \in \mathbb{R}^T$, that can be interpreted as prices associated with an imbalance between energy exported to and imported from the REC pool, represented by the global constraints $h(\Theta)$. In fact, π is the Lagrange multiplier associated with the constraints $h(\Theta)$.

We have the following property.

P6. *All individual constraint sets $\overline{\Omega}_i$ and the joint strategy set $\mathcal{C}$ satisfy the Slater's constraint qualification (see Section 3.1.3, [94, (5.27)]).*

The connection between the solutions of the GNEP $\mathcal{G}$ and the NEP $\mathcal{G}_{ext}$ is displayed in the Lemma B.1 [183].

Lemma B.1 (Extended game)**.** *A point $(\Theta^*, \pi^*) \in \mathcal{C} \times \mathbb{R}^T$ is a Nash equilibrium of the game $\mathcal{G}_{ext}$ if and only if Θ^* is a solution of the VI$(\mathcal{C}, F)$ with multiplier π^* associated with the shared constraints $h(\Theta^*) = 0$.*

We have transformed the VEs computation of the GNEP (4.27) into solving an extended NEP (B.5). The NEs of $\mathcal{G}_{ext}$ coincides with the solutions of an extended VI problem that can be achieved via distributed algorithms.

Lemma B.2 (Extended VI)**.** *A point $(\Theta^*, \pi^*) \in \mathcal{C} \times \mathbb{R}^T$ is a Nash equilibrium of the game $\mathcal{G}_{ext}$ if and only if it is a solution of the VI$(\mathcal{Y}, F_{ext})$, with $\mathcal{Y} := (\prod_{i \in \mathcal{N}} \overline{\Omega}_i) \times \mathbb{R}^T$ and*

$$F_{ext}(\Theta, \pi) := \begin{bmatrix} (\nabla_{\Theta_i} b_i(\Theta_i, \Theta_{-i}) + \nabla_{\Theta_i} \pi^\top h(\Theta_i, \Theta_{-i}))_{i=1}^N \\ -h(\Theta) \end{bmatrix} \tag{B.6}$$

We summarize the previous results with Theorem B.1.

Theorem B.1. *A point Θ^* is a variational equilibrium of the GNEP (4.27) if and only if a π^* exists such that (Θ^*, π^*) is a solution of the extended*

VI($\mathcal{Y}, F_{ext}$).

We have decoupled the members' constraints by incorporating the global constraints $h(\Theta)$ into the objective function. Thanks to the reformulation in Theorem B.1, we obtain the set $\mathcal{Y}$ as the Cartesian product of individual feasible sets. Hence, as in A.2, we solve a regularized sequence of VI($\mathcal{Y}, F_{ext} + \tau(I - (y^k, \eta^k))$) with (y^k, η^k) in $\mathbb{R}^{n+T}$.

Convergence of the algorithm is studied in Theorem B.2.

Theorem B.2. *Let* $\mathcal{G} = (\mathcal{N}, \mathcal{C}, (b_i)_{i=1}^N)$ *be a GNEP as in* (4.27) *with a nonempty set solution. If*

1. *the regularization parameter* τ *satisfies* $\tau > \tilde{\tau}$*, such as:*
   - *if tariff T1 is in effect, for [EB]*

$$\tilde{\tau} = \frac{2\alpha(N-1)}{N} + 2.\sqrt{\frac{\alpha^2(N-1)^2}{N^2} + N} \tag{B.7}$$

   and for [CB]

$$\tilde{\tau} = \alpha.(N-1) + \sqrt{\alpha^2.(N-1)^2 + 4N}. \tag{B.8}$$

   - *if tariff T2 is in effect, for all cost distribution methods*

$$\tilde{\tau} = \sqrt{2N}. \tag{B.9}$$

2. ρ *is chosen such that* $\rho \subset [R_m, R_M]$*, with* $0 < R_m < R_M < 2$*,*

then, any sequence $\{(\Theta^k, \pi^k)\}_{k=1}^\infty$ *generated by PDA converges to a variational equilibrium of the GNEP.*

The convergence proof of the PDA with shared constraints is based on the relation between the extended NEP and VIs. We check the result for a GNEP under T1 tariff and continuous billing. The other cases follow a similar reasoning. The proof follows the lines of argument in [124, 182, 184]. Recalling Lemma B.2, solving the extended NEP $\mathcal{G}_{ext}$ in (B.5) is the same as solving the VI($\mathcal{Y}, F_{ext}$), with $\mathcal{Y} := (\prod_{i\in\mathcal{N}} \overline{\Omega}_i) \times \mathbb{R}^T$ and

$$F_{ext}(\Theta, \pi) := \begin{bmatrix} F(\Theta) + \pi^\top \nabla_\Theta h(\Theta) \\ -h(\Theta) \end{bmatrix}. \tag{B.10}$$

Then, we solve a regularized sequence of $\mathrm{VI}(\mathcal{Y}, F_{ext} + \tau(I - (y^k, \eta^k)))$ with (y^k, η^k) in $\mathbb{R}^n \times \mathbb{R}^T$. According to [122, Th. 17], we need to show that: (a) the mapping F_{ext} in (B.10) is monotone on $\mathcal{Y}$; (b) the regularization parameter τ is large enough such that the $N+1$ square matrix $\overline{\Upsilon}_{F,\tau}$ is a P-matrix

$$\overline{\Upsilon}_{F,\tau} := \begin{pmatrix} \Upsilon_F + \tau I_N & -\mu \\ -\mu^\top & \tau \end{pmatrix} \tag{B.11}$$

$$\mu := \Big(\max_{\Theta_i \in \overline{\Omega}_i} \|\nabla_{\Theta_i} h_i(\Theta_i)\|_2 \Big)_{i=1}^{N} \tag{B.12}$$

with Υ_F defined in (A.4)-(A.6).

Proof of (a). If F is monotone on $\prod_{i\in\mathcal{N}} \overline{\Omega}_i$, so is F_{ext} on $\mathcal{Y}$ [182, Prop. 4.4]. Since the matrices are similar to those obtained with model D1 in Appendix A.2, we can directly conclude that F is monotone. ■

Proof of (b). We determine the value of τ such as $\overline{\Upsilon}_{F,\tau}$ is a P-matrix. In fact $\overline{\Upsilon}_{F,\tau}$ is a Z-matrix (i.e., all off-diagonal elements are non-positive). We write $\overline{\Upsilon}_{F,\tau} \geqslant \tilde{\Upsilon}_{F,\tau}$, where $\geqslant$ indicates component-wise $\geqslant$, and

$$[\tilde{\Upsilon}_{F,\tau}]_{ij} := \begin{cases} \tau & \text{if } i = j \\ -2\alpha & \text{if } i \neq j \text{ and } i, j \neq N+1 \\ -2 & \text{otherwise} \end{cases} \tag{B.13}$$

If $\tilde{\Upsilon}_{F,\tau}$ is a Z and P-matrix, then $\overline{\Upsilon}_{F,\tau}$ is P-matrix [185, Thm. 3.11.10]. We have that $\tilde{\Upsilon}_{F,\tau}$ is a P-matrix if and only if the spectral radius of the matrix $\Gamma_{F,\tau}$ is less than 1 [122]:

$$[\Gamma_{F,\tau}]_{ij} := \begin{cases} 0 & \text{if } i = j \\ \upsilon_{ij}^{\max}/\tau & \text{if } i \neq j \text{ and } i, j \neq N+1 \\ \mu/\tau & \text{otherwise} \end{cases} \tag{B.14}$$

where the spectral radius is the maximum of the absolute values of its eigenvalues. According to the Gershgorin circle theorem, every eigenvalue of $\Gamma_{F,\tau}$ is contained in at least one of the Gershgorin disks $D(0, R_i)$ with $R_i = \sum_{j\neq i} |[\Gamma_{F,\tau}]_{ij}|$. Hence, $\overline{\Upsilon}_{F,\tau}$ is a P-matrix if for some $\omega > 0$, the conditions

$$\tau > 2\alpha(N-1) + 2\omega \tag{B.15}$$

$$\tau > \frac{2N}{\omega}, \tag{B.16}$$

are verified. The ω value minimizing τ is a solution of the second-degree equation

$$2\alpha(N-1) + 2\omega = \frac{2N}{\omega} \Leftrightarrow 2\omega^2 + 2\alpha(N-1)\omega - 2N = 0. \tag{B.17}$$

By incorporating this data into (B.15), we have

$$\tau > \alpha(N-1) + \sqrt{\alpha^2(N-1)^2 + 4N}. \tag{B.18}$$

$\blacksquare$

Remark B.1. The case of [NET,VCG], under tariff T1, is more sensitive. We calculate a regularization parameter τ larger enough such that the $N+1$ square matrix $\overline{\Upsilon}_{F,\tau}$ is a P-matrix. Let $i, j \in \mathcal{N}$, if $i \neq j$, we have $[\tilde{\Upsilon}_{F,\tau}]_{ij} = -4\alpha \max_i K_i$ and so

$$\tau > 2\alpha(N-1)\max_{i\in\mathcal{N}} K_i + 2\sqrt{\alpha^2(N-1)^2 \max_{i\in\mathcal{N}} K_i^2 + N}.$$

However, the function F_{ext} is not monotone, but it is a P_0-function on $\mathcal{Y}$ which is closed and convex. Furthermore, the VE set is non-empty and compact, and in our use-case we observe convergence with low inefficiency (see Section 4.6.2 on page 106 and Table 4.6 on page 113).

Even though $\mathcal{G}$ is a potential game, the structure of its set of strategies is non-Cartesian, so that we cannot use the same arguments as in the NEP case in Remark A.1. Nevertheless, we can apply the method of [118], being aware that convergence to a VE is not guaranteed, although the generated sequence is on $\mathcal{C}$ and each limit point is a GNE. In fact, there are no proofs or indications concerning the type of equilibrium (VE or not) one could get depending on the initial inputs. In [NET,VCG] case, there is no theoretical guarantee that VEs offers the cost optimal value, but we have Theorem 4.5.1.

APPENDIX C.

SPE outcomes

C.1. REC in deficit

Criteria		Num.	User	PV	Stor.	Local	Exp. utility	Exp. utility
Users $\mathcal{M}$	REC	SPE		[kWp]		prices	user	REC
NPV1	NPV1	1	7	+1	0	D	11 695.05€	-124 731.46 €
NPV2	NPV2	1	7	+1	0	D	17 435.87€	-183 098.96 €
C_{tot}	C_{tot}	1	7	+1	0	D	-18 159.8€	190 482.01€
ROI	(financial)	10	7	+1-+10	0	D	-0.376%	
ROI	CDE	1	2	+10	0	I	0.032%	104.79 tCO2eq
(financial)	PkWh*	243	7	+0	0	I		0.185 €/kWh
NPV1	CDE	1	7	+1	0	D	11 695.05€	105.59 tCO2eq
NPV2	CDE	1	7	+5	0	C	17 938.6€	105.227 tCO2eq
C_{tot}	CDE	1	7	+10	0	I	-19 204.31€	104.79 tCO2eq

Table C.1.: Expected utilities of SPEs obtained for the NMIP Γ^2 of the REC in deficit with the candidates set $\mathcal{M}$ and the rationality of agents is assumed. The (financial) notation indicates that the results are valid if one of the three criteria: NPV1, NPV2 and C_{tot} is used. We have used the lexicographical order in the case of PkWh.

C.2. REC in surplus

Criteria		Num.	User	PV	Stor.	Local	Exp. utility	Exp. utility
Users $\mathcal{M}$	REC	SPE		[kWp]		prices	user	REC
NPV1	NPV1	1	11	+0	0	I	-106 859.6€	-31 048.7 €
NPV2	NPV2	1	11	+0	0	I	-153 351.6€	-47 560€
C_{tot}	C_{tot}	1	11	+0	0	I	159 213.19€	49 660.1€
CDE	CDE	1	11	+10	0	C	25.575 tCO2eq	68.478 tCO2eq
ROI	(financial)	10	11	+0	0	I	0%	
ROI	CDE	1	7	+1	0	I	-0.175%	67.610 tCO2eq
(financial)	PkWh*	1	7	+1	0	I		0.101 €/kWh
NPV1	CDE	1	7	+1	0	I	11 895.42€	67.610 tCO2eq
NPV2	CDE	1	7	+1	0	I	17 727.52€	67.610 tCO2eq
C_{tot}	CDE	1	7	+1	0	I	-18 462.97€	67.610 tCO2eq
CDE	(financial)	21	11	+0	0	I	33.788 tCO2eq	
PkWh*	(financial)	231	11	+0	0-+1	I	0.17 €/kWh	

Table C.2.: Expected utilities of SPEs obtained for the NMIP Γ^2 of the REC in surplus with the candidates set $\mathcal{M}$ and the rationality of agents is assumed. The (financial) notation indicates that the results are valid if one of the three criteria: NPV1, NPV2 and C_{tot} is used. We have used the lexicographical order in the case of PkWh.

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